

WHAT YOUR COLLEAGUES ARE SAYING . . .

Rachel Lambert skillfully delivers cogent background and advice for addressing the complexity of teaching mathematics to students with disabilities. By contextualizing all approaches in an environment of high expectations, trust, and strengths-based instructional approaches, she suggests a carefully crafted continuum to spur one to rethink what mathematics educators and special educators do. Every individual engaged in working with K–8 learners needs to delve into this critical information. Let the sense-making begin.

Karen Karp

Professor, John Hopkins University
Baltimore, MD

This book by Rachel Lambert, a leading expert in special education and mathematics education, is an invaluable contribution to our thinking and knowledge of the best ways to support students with learning differences in their understanding of mathematics. A must-read for any educator or parent.

Jo Boaler

Nomellini-Olivier Professor, Stanford
Stanford, CA

This book is long overdue! I applaud Lambert for challenging mathematics educators to reimagine what students with disabilities are capable of in mathematics. Lambert confronts the deficit perspectives that have permeated the way many educators think about students with disabilities in mathematics. Lambert also provides specific examples that show that students with disabilities can engage in rich, cognitively demanding mathematics when given the proper support and guidance.

Kyndall Brown

Executive Director, California Mathematics Project
Los Angeles, CA

All too often in my career, I've heard neurodiverse students discussed in deficit ways in terms of their opportunities to know and do mathematics. Lambert's book provides shining examples of strategies we can use to empower and provide access to neurodiverse students so that they can demonstrate their full mathematical brilliance in math class. The strategies described are simultaneously revelatory and straightforward to implement.

Michael D. Steele

Professor, Ball State University
Muncie, IN

UDL can now truly become a tool that mathematics educators can use to change how they go about designing mathematics experiences for disabled students.

John Staley

Past President, NCSM
Coordinator, Baltimore County Public Schools
Randallstown, MD

This book is a transformer! *Rethinking Disabilities and Mathematics* is a must-read for all educators. Dr. Lambert provides insight into different types of disabilities and strategies to implement UDL Math to engage and support learners in developing positive mathematics identities. She goes beyond strategies and instruction, giving us clarity through the lens of different perspectives that will make educators rethink disability.

Joleigh Honey

Past President, Association of State Supervisor of Mathematics
Author/Consultant, OpenUp Resource High School Mathematics
Executive Committee, Mathematical Sciences (CBMS)
Sandy, UT

As I read this extraordinary book, it felt like I was having a conversation with Rachel Lambert. Connecting with my innermost being in a deep and meaningful way; this is what is needed to change mindsets and practices. The vignettes, coupled with well-documented research, allow the reader to reflect on their own experiences and teaching schema with practical transformative methods from the perspective of not only researchers but also authentically from teachers.

Christina Lincoln-Moore

Elementary Mathematics Coordinator,
Los Angeles County Office of Education
Downey, CA

The word “strength” rarely makes its way into conversations about students’ disabilities in school buildings. In Lambert’s work, strength is a basic tenet of students with disabilities. From classroom visits to conversations with mathematicians with disabilities, Lambert unveils a landscape of possible math learning with a UDL Math framework and strengths-based pedagogies because disabled learners were never the problem to be fixed in math education, the system is.

Fawn Nguyen

Math Teaching and Learning Specialist, Amplify
Oak View, CA

If there is one book you need to read about the field of special education and mathematics, this is it. Rachel Lambert has torn apart our old tapestry of beliefs and teaching practices and rewoven another with clarity, powerful examples, and research. This book breaks new ground.

Catherine Fosnot

Professor Emerita of Education, City College of New York
Founder, Mathematics in the City
New York, NY

This is a groundbreaking exploration that transcends traditional teaching paradigms and revolutionizes our perspectives on disability in mathematics education. With insightful chapters exploring the brilliance of diverse learners, real-life stories, and rigorous research on UDL, it empowers educators with a strengths-based approach. A transformative guide, it ignites introspection and catalyzes action, shaping a more inclusive mathematical landscape.

Ma Bernadette Andres-Salgarino

Assistant Director, iSTEAM, SCCOE
President, CMC (2024–2025)
San Jose, CA

This is the book I've been hoping for for so long! Lambert dares us to think radically about and reimagine math instruction and disability, while also giving us the practical, classroom-tested tools to bring these bold dreams to life. I feel both inspired and equipped to make real progress in my district with this trusted guide at my side.

Tracy Johnston Zager

Math Coach
and Author of *Becoming the Math Teacher You Wish You'd Had*
Portland, ME

Rethinking **DISABILITY and MATHEMATICS**

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To my father, who taught me that normal was boring.
To Dina Williams, who welcomed me to the math community
in Los Angeles.

Rethinking **DISABILITY** and **MATHEMATICS**

A UDL Math Classroom Guide for Grades K-8

RACHEL LAMBERT

Foreword by Megan Franke

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1 Oliver's Yard
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United Kingdom

SAGE Publications India Pvt. Ltd.
Unit No 323-333, Third Floor,
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Place
New Delhi – 110 019
India

SAGE Publications
Asia-Pacific Pte. Ltd.
18 Cross Street #10-10/11/12
China Square Central
Singapore 048423

Vice President and Editorial Director:
Monica Eckman
Acquisitions Editor: Debbie Hardin
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Note From the Publisher: The author has provided video and web content throughout the book that is available to you through QR (quick response) codes. To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

FOREWORD

By Megan Franke

A number of years ago I was involved in a large-scale study investigating how teachers could use research about the development of students' algebraic thinking to support their classroom practice and student learning. As a part of the study, we individually assessed six randomly selected students in third- through fifth-grade classrooms. When I approached one of the fifth-grade teachers to share the list of students who had been randomly selected, she pointed to the name of one student and told me that I did not want to assess Jayden because he was in special education and struggled with mathematics. I asked if it would be okay if I started our assessment and if Jayden seemed uncomfortable or could not engage the tasks, I would end the assessment and bring him back to class. The teacher agreed.

I discovered that Jayden's solutions were mathematically sophisticated and beyond what we heard from most of the other students I assessed at the school. He could engage in what we term *relational thinking*—when asked to solve a problem like $47 + 35 = ___ + 45$ he could say that 45 is 2 less than 47 so the answer has to be 2 more than 35. He thought like this consistently throughout the assessment. I sometimes had to ask him to repeat himself as the numbers he was saying seemed to fly out of his mouth so quickly that I was not quite sure how he was solving the problem and needed to hear it again; but every time, he explained he used relational thinking in mathematically elegant ways. It is not surprising that those working with him at school did not know that he could think in these mathematical ways. School would be teaching Jayden to use the standard algorithm to add 47 and 35 and then subtract 45. Teachers would be working hard to get him to learn it this way, which likely would lead to confusion for Jayden because it is not how he was thinking about the solution to the problem. In fact, how Jayden was thinking about the problem was more efficient, mathematically more sophisticated, and better connected to what he needed for his future mathematical work in algebra.

Schools often do not allow space for students like Jayden to share what they know or solve math problems in ways that make sense to them. This is a challenge of schooling, not of a single teacher, but a challenge that extends to how teaching and learning occurs in classrooms. We expect all students to think the same way, solve problems the same way, use tools in the same

FOREWORD

way, and all at the same time. Doing so means school only works for certain students—students who think the way the curriculum is asking, are thinking that way when the teacher lands on that page, and they can share what they know in the way the curriculum is asking. This approach provides an extreme challenge for students who work differently, process differently, share what they know differently, and so on. It also provides a challenge for the teachers who work with these students.

In *Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K–8*, Dr. Lambert asks us to disrupt this approach to schooling in mathematics and prompts us instead to see the breadth and depth of students' mathematical capabilities. Students may not know the math that school wants them to know or be able to show what they know in the way school expects, but they know valuable mathematics—such as how to build a Lego structure, draw a picture of their family, or organize their collection of toy cars by type and color. They may know that 4 quarters is a dollar and that means 4 one-fourths is one whole or that you can double a number and then double it again and again and again. Seeing and hearing what young people bring to math class feels like a tall order when working with students with disabilities. I often hear the worry that students with disabilities are behind and cannot participate in the ways we are asking of them. Many educators note that it can be hard to figure out what they know. We worry that we will ask too much of the students and they will get quickly frustrated. In a well-intentioned effort to help students, we reduce the options (fewer tools, less complexity in the task, more practice) and show students what to do. However, reducing options leads us to underestimate students and provide them with little agency. Dr. Lambert provides a vision and practical ways of enacting that vision that move beyond typical schooling responses by centering students and helping us see how to build on what they know and can do.

Dr. Lambert challenges all of us to consider what it means to see, hear, and support students with disabilities in our mathematics classrooms. She asks us to reflect on our views of students with disabilities and their capabilities. She helps us see how much schooling shapes how we think about both what counts as student success and what it means to do mathematics. Yet, she also provides a vision of what classroom mathematics can look like that supports students with disabilities. She provides many real-life vignettes that get to the heart of interaction in math classrooms and what it would look and feel like to support a range of student participation, enable students to share what they know, allow them to take the lead mathematically, and do so in inclusive classrooms.

Brilliance, patience, support, and differential participation come up over and over throughout this book. But so does the reality of schooling for students with disabilities and the challenges that emerge. Dr. Lambert brings forth the voices of teachers and students in classrooms to help all of us see what it means to support students with disabilities and reminds us that supporting students with disabilities means understanding the assets the student brings along with developing particular strategies for supporting them. Dr. Lambert repeatedly reminds us that our challenge as educators is to move beyond seeing what students know as isolated and random ideas but as knowledge that draws on valuable spatial, number, and grouping (and so on) understandings. This groundbreaking book will help educators recognize what students know and can do mathematically as an important part of the development of their mathematical thinking and ultimately what we want them to learn in school mathematics.

PREFACE

I wrote this book because of a puzzle I had as a classroom teacher. I taught for 6 years at River East Elementary School in East Harlem, New York City. It was a progressive, teacher-run school that gave teachers the freedom to develop curriculum. I was initially a special education co-teacher and then took over the classroom teaching for a multi-age, inclusive fifth/sixth-grade classroom. I noticed again and again that students came into my class telling me they were “bad at math” and then began to love the subject once they were allowed to think. I found this particularly true for the students with disabilities in my classroom.

My puzzle emerged once I started reading research on special education and math. I was shocked by what I found. Article after article focused on deficits. Some articles explicitly claimed that students with disabilities should not be exposed to inquiry mathematics and implicitly said that these students did not know how to think on their own.

I knew this was wrong. As a person who had grown up with neurodiversity in my family, friends, church, and community, this deficit attitude toward the potential of neurodiverse and disabled people angered me. This deficit attitude particularly angered me since my students were not only in special education but also Black and Latino/a, and thus they should be offered more educational opportunity, not less. The research seemed to contradict everything I was experiencing as my students grew in their confidence as mathematicians.

This puzzle motivated me to leave the classroom after 10 years and become an educational researcher in meaningful mathematics for students with disabilities, with a particular focus on neurodiverse students of color. My problem of practice has been to better understand the experiences of disabled and neurodiverse students learning mathematics through sense-making.

This book is an existence proof; students with disabilities can and do become confident mathematical thinkers. Teachers all over the world are doing the hard work of making this happen. I am here to share these stories. This book has grown out of 15 years of research, as well as out of continued work in

schools doing professional development. Teachers from research projects, whose identity I need to protect as part of the terms of research, are given pseudonyms (i.e., Mr. Jay and Ms. Rey). Teachers from my professional development work are referred to by their first names. All students, whether I met them in a visit to a class or in a research project, are given pseudonyms. I use line drawings taken from photographs to show how students and teachers engaged with each other while preserving anonymity.

NOTES ON LANGUAGE

Writing a book about disability means dealing with complexity in terms of language. First, for advocates in the Disability Rights Movement and academics in disability studies, the word *disability* is not a negative word but a word that is rich with political power. I often get asked why I don't use *differently abled*, *special needs*, *dis/ability*, or some other "nicer" term. These alternative terms are almost always created by nondisabled adults to refer to children with disabilities. Some disabled adults find these kinds of euphemisms insulting, so I do not use them.

I use both person-first language and identity-first language in this book. Person-first language puts the person first and the disability second, such as *person with autism*. This approach is often preferred by those with intellectual disabilities (*a person with an intellectual disability*). Other disability communities tend to prefer identity-first language, such as *autistic person*. Increasingly, identity-first language such as *disabled person* is used by disabled people with pride as it emphasizes how disability does not reside within the individual, but in the environment. I use both kinds of language in this book to respect multiple points of view. Just as in any other social justice movement, people have the right to name themselves, and I try to respect that with my language. While at times in this book I am writing about the many different categories that we currently understand as disability, including sensory disabilities or mobility disabilities, I primarily focus on neurodiversity in this book. At times I discuss disability and neurodiversity, and at other times I use disability as an all-encompassing word that includes neurodiversity.

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Further thanks go to colleagues at the California Mathematics Project, particularly those at the University of California, Santa Barbara (UCSB) Math Project. Also, my colleagues at the University of California, Los Angeles Math Project who have accepted me into the math community here in Los Angeles.

Much of what I share here came from teaching at River East, a small teacher-run progressive elementary school in East Harlem, New York. I am forever grateful for my innovative colleagues there, as well as for the children and families who taught me so much.

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Finally, I want to thank my family growing up, who taught me about neurodiversity. I owe a lot to what I learned from my friend Ali. I am grateful beyond words for Kevin, Rufus, Jack, and Ruby who supported me to write, write, write!

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K-5 Math Coach, UCLA Mathematics Project
Redondo Beach, CA

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Associate Professor of Mathematics Education, University of Mississippi
Houston, MS

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Past President, Association of State Supervisors of Mathematics
Author & Consultant, OpenUp Resources High School Mathematics
Executive Committee, Conference Board of the Mathematical
Sciences (CBMS)
Sandy, UT

Rosa Serratore
PreK–12 district mathematics coordinator
NCSM regional director
CMC board member
Santa Monica, CA

John William Staley
Past President NCSM
Coordinator, Baltimore County Public Schools
Randallstown, MD

Michael D. Steele
Professor, Ball State University
Zionsville, IN

Catherine Anne Vittorio
Principal, Etiwanda School District
Etwinda, CA

ABOUT THE AUTHOR



Rachel Lambert is an associate professor in special education and mathematics education at the University of California, Santa Barbara. Before becoming a professor, she worked for more than 10 years as both a special education and a general education inclusion teacher in New York City, San Francisco, and Los Angeles. Dr. Lambert researches how students of color with disabilities construct identities as math learners and the role of emotions in mathematics. She also researches UDL in the area of mathematics, as well as neurodiversity in mathematics. Her goal is to increase access to meaningful mathematics for students with disabilities.

CHAPTER 1

TRUST IN THEIR THINKING

IN THIS CHAPTER, WE WILL . . .

- Visit a fifth-grade special education classroom in which students are engaged in meaningful mathematical inquiry
- Provide an overview of this book

Trust has been my key to becoming an effective math teacher. I trust that my students have the ability to make sense of math. I trust that they will arrive at the understandings they need at their pace. Trusting my students has empowered them. In my classroom, students know that I will not be coming to the rescue at the first sign of trouble because I believe in their own abilities. When they get stuck I redirect them back to themselves, “What do you know about numbers that can help you?”

“It’s hard,” they tell me. “Yup, but you’ll figure it out,” and eventually they do. They do because through my trust, they trust themselves. We need to empower students to trust themselves.

Figure 1.1 • Sussan De Matta, fifth-grade special education teacher



—Sussan De Matta, fifth-grade special education teacher (Figure 1.1) ●

I begin this book with this idea, expressed by this exceptional math teacher, that when we as teachers trust in the thinking of our students, especially our students with disabilities, we create the conditions our students need to trust themselves as mathematical thinkers. When we, their teachers, believe that our students can and will solve complex problems, they will be able to.

In my work for almost 30 years in education, first as a general education and special education classroom teacher and now as a teacher educator and researcher, I have seen students with disabilities underestimated and over-scaffolded. These students are conceptualized as broken, as needing to be fixed. Their mathematical thinking is not trusted. Deficit conceptions of learners turn into deficit pedagogies that assume students cannot think for themselves. And these problems become intensified for students with disabilities who are Black, Latino/a, Indigenous, and/or multilingual.

This book is dedicated to overturning deficit pedagogies and returning mathematical agency to all students with disabilities. Mathematics can and should be a transformative space for these students, where they can discover their power and potential.

In this book, you will be invited into classrooms like Sussan's that provide students the space and support to allow students with disabilities to thrive as mathematical sense-makers. We will meet teachers who believe that all their students are mathematical thinkers and who design classrooms to build on students' thinking. We will highlight the mathematical thinking and brilliance of students with disabilities. We will frame these classrooms using Universal Design for Learning in Mathematics (UDL Math), which applies the theoretical framework of UDL to research in meaningful mathematics.

Before we jump into Sussan's classroom, you need to do some math! In order to appreciate the mathematical brilliance of the children, you need to tackle the problem first. So grab some scratch paper and get started. I recommend starting off, as children most often do, by drawing.

Try It

1. I have 12 cans of paint. I need $\frac{1}{4}$ th of a can of paint to paint one chair. How many chairs can I paint?
2. What if it takes $\frac{3}{4}$ ths of a can of paint to paint one chair? How many chairs can I paint with 12 cans of paint?

What did you notice about this problem? It's a tricky one, for sure. Some might start by drawing paint cans and chairs and then distributing $\frac{1}{4}$ th of each can of paint to each chair. Some might find a ratio and work from there. Maybe some might think: If $\frac{1}{4}$ th of a can of paint will cover 1 chair, then $\frac{4}{4}$ ths of a can of paint will cover 4 chairs, and then they could

multiply 4×12 to get the total. Some might have added or skip-counted the $\frac{1}{4}$ ths until they got to 48. In that case, you might have seen this situation as

$$\left| \quad \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 48 \right.$$

Or perhaps as $12 \div \frac{1}{4} = 48$, thinking about it as how many groups of $\frac{1}{4}$ ths fit into 12?

Although this is a division of fractions problem, that is often not initially obvious to either children or adults. The first two strategies I described use multiplication to figure out how many groups of $\frac{1}{4}$ th. For a problem like this, which Susan Empson and Linda Levi (2011) call a *Multiple Groups Problem (Division)*, kids usually draw it out to understand the relationships involved. I am not embarrassed to say that I do the same when I am multiplying or dividing with fractions. I always need to see the situation in a model before I understand exactly what is happening. In this way, I am like most students, who begin with *direct modeling*, a term from cognitively guided instruction (CGI, Carpenter et al., 2015) that simply means that a student chooses to represent each item in a problem to solve it, either with manipulatives or through drawing.

This kind of fraction division problem is included in the fifth grade in the Common Core State Standards (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010)—students are asked to solve real-world fraction division problems like the one in this case, a whole number divided by a unit fraction ($12 \div \frac{1}{4}$). The standards set the expectation that students will begin by using visual models to solve these problems, drawing on their knowledge of both fractions and how multiplication and division are related. As we will see, Sussan offered her special education students the choice of solving with either $\frac{1}{4}$ ths or $\frac{3}{4}$ ths, the first option meeting grade-level standards and the second number choice (a common fraction with a number other than 1 in the numerator) going beyond them.

SUSSAN'S FIFTH-GRADE SPECIAL EDUCATION CLASS

Sussan taught a fifth-grade special education class, in which all the students received special education services. The students in Sussan's class had IEPs for either learning disabilities or autism, with multiple students also receiving

speech and language services. Her students were primarily Latino/a, and several were multilingual learners. She teaches all subjects to the students, who move in and out of the room as some attend general education for certain subjects.

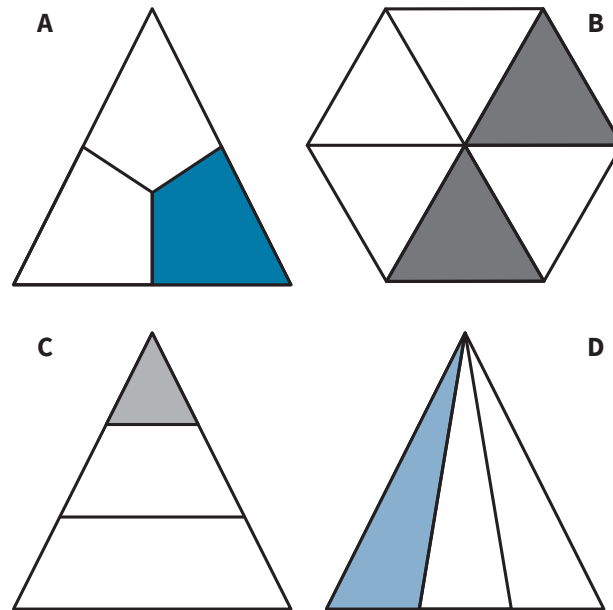
Through multiple conversations with Sussan, I learned that her ultimate goal in her mathematics teaching is for her students to see themselves as successful mathematics learners, to be able to solve problems and persevere, to engage in mathematical discussions, and to reason quantitatively. Beginning with grade-level standards, she flexibly designs her curriculum based on what her kids are currently thinking and doing, using her professional development work in CGI with her district to help her notice and build on student thinking. Throughout, she has a focus on helping her students become strategic learners.

I VISITED HER class with a group of special educators from her district who were interested in teaching mathematics through problem-solving. Sussan started with a warm-up in which she showed four images of fractions and asked students the question, “Which One Doesn’t Belong?” (which she found on the website Fraction Talks [<https://bit.ly/3EVJJG6>]). She started by giving a little independent think time; then the students talked in pairs about what they noticed; and then they shared with the whole group. She annotated the image, using a tablet that was connected to the classroom projector. She added both the words students used and the way they named the fractions (Figure 1.2).

Try It

Which one doesn’t belong? Why? Can you make an argument for each shape in Figure 1.2?

Discussion of this image was short, no more than 7 minutes, but it was a lovely way to get ready for a longer fraction problem. A student volunteered the upper right image of a hexagon (B) as the one that did not belong, as it was a different fraction, $\frac{2}{6}$ s versus $\frac{1}{3}$ rd. Other students disagreed, arguing that since those fractions were equivalent, this one did belong! When one student proposed that the bottom left triangle (C) did not belong, others nodded. This

Figure 1.2 • Which One Doesn't Belong image

SOURCE: Reprinted with permission from Nat Bantling at FractionTalks.com. Image created by Denise Gaskins.

was the only fraction that was not equal to $\frac{1}{3}$. One student argued that it wasn't a fraction at all because it was "uneven." Sussan designed this discussion as a warm-up and as a chance to use some vocabulary around fractions (equivalence, equal shares). I noticed that these students had a strong grasp of the fundamentals of fractional equivalence.

Next, Sussan moved to the story problem that was the main focus of the class. She started by introducing students to the context of painting chairs, showing a funny GIF about painting. She asked the students what they knew about painting at home. She gestured painting a chair while she showed multiple images of painting a chair and cans of paint. Students talked in pairs about what they knew about painting. When she felt sure they all understood the context of painting furniture using cans of paint, she introduced the 12 chairs problem. She showed the following slide, reading the text out loud twice. While keeping 12 chairs constant, she gave two number choices, either $\frac{1}{4}$ th or $\frac{3}{4}$ ths a can of paint for each chair.

Your parents have asked you to help them paint some chairs. You have 12 cans of paint. If you need _____ cans of paint to paint a chair, how many chairs can you paint?

$$\left[\frac{1}{4} \text{ or } \frac{3}{4} \right]$$



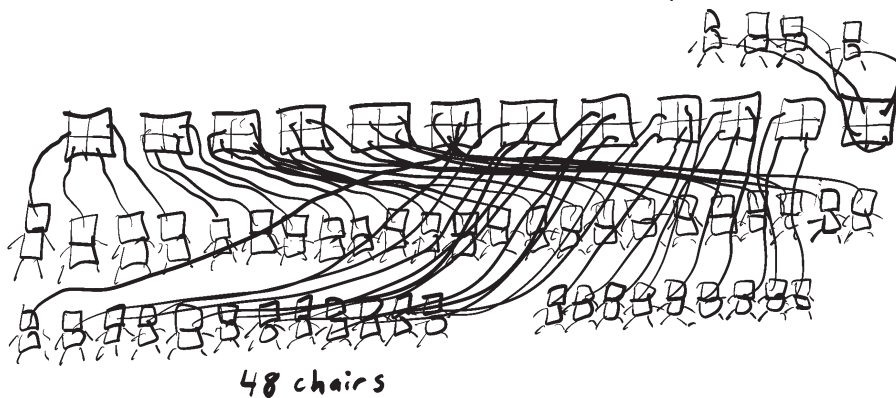
SOURCE: istock.com/anchiy

As the students went off to work, scattering around the classroom into desks, or sprawling out on the carpet, Sussan and her two paraprofessionals supported students. One paraprofessional helped a student who wanted to use manipulatives. This drew me over to watch as I am always fascinated/frightened when kids use manipulatives for fractional share problems. Take a moment and think about how you might use Cuisenaire rods to solve this problem. Confusing? Yes. Using manipulatives means that the kids need to assign a fractional value to a piece, and then make sense of the problem, almost creating another layer to their proportional thinking. I noticed that the paraprofessional did not dissuade the student, who counted out 12 rods, looked at them, looked at his paper, and then changed his mind and drew out the problem instead. Another paraprofessional was supporting a student who asked her to scribe for her. As the student verbally explained her strategy, the paraprofessional wrote down her words.

I became fascinated watching one girl, who we will call Amaris, who drew 12 boxes, which represented the cans of paint. She partitioned them into fourths

and then seemed to be stuck. After a moment or so looking at the paper, she started drawing chairs (a reconstruction of her work can be seen in Figure 1.3). She meticulously drew each chair as she connected each chair to a $\frac{1}{4}$ th of a can of paint. She eventually drew each of the 48 chairs necessary for her number choice. It was quite something to watch her. I looked around and saw four other teachers gathered, all of us seemingly spellbound by her work (Figure 1.3).

Figure 1.3 • *Amaris's strategy for 12 chairs and $\frac{1}{4}$ th a can of paint*

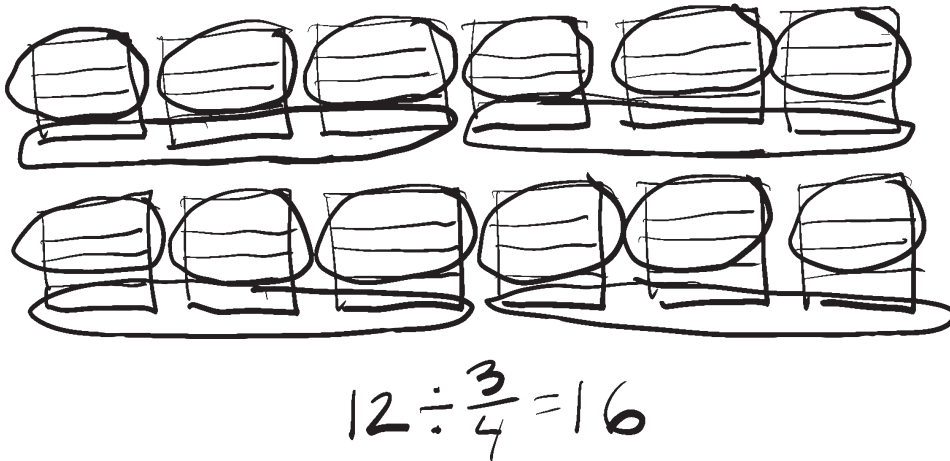


Later, I asked Sussan if Amaris did this kind of modeling every time. Sussan told me that in Amaris's previous classrooms, she had not been given access to solving problems on her own, and so it had taken her a few months to feel comfortable making sense of a problem. At the beginning of the year, she would ask Sussan to tell her how to solve every problem. Again and again, Sussan encouraged her to draw it out or to use manipulatives. It took time and a lot of patience for both teacher and student, but Amaris was now learning how to direct model through drawing. It took time for her to trust herself, and we were honored to be able to see her just beginning to trust that she could figure it out herself.

Sussan walked around the room, observing students quietly from a little distance. At times, she would bend down to a child and ask a few questions about what they were thinking. Her presence communicated a very quiet support, a sense that she believed these students could solve this complex problem.

Although most kids in the class choose $\frac{1}{4}$ th, one child, Marty, immediately chose $\frac{3}{4}$ ths as his number choice. He solved it by drawing the cans of paint, partitioning them into fourths, and grouping the fractional parts into $\frac{3}{4}$ ths (Figure 1.4).

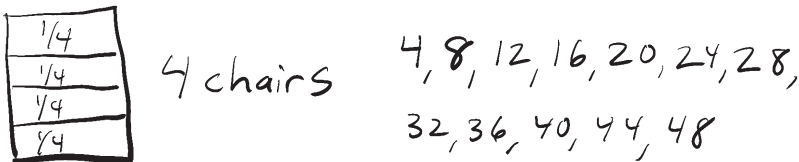
Figure 1.4 • Marty's strategy for 12 chairs and $\frac{3}{4}$ th a can of paint



I noticed that Marty immediately wrote an equation to describe the problem, correctly identifying the problem as division.

Another student, Erica, chose $\frac{1}{4}$ th. She partitioned one can of paint, finding that one can of paint worked for four chairs. She then skip-counted by four 12 times to get her answer (Figure 1.5).

Figure 1.5 • Erica's strategy for 12 chairs and $\frac{1}{4}$ th a can of paint



As the class gathered for a share on the rug, Sussan joined their circle, arm in a brace. She first had all the students turn and talk, explaining their strategy to a partner. She listened in on these shares. Afterward, Susan asked Marty to share. As he shared, she rewrote his strategy on her tablet, which was projected onto a screen in the front of the room. He did so confidently, sharing

his thinking with little prompting from his teacher. After he had shared through his process once, she had him talk through it again, stopping at several points for students to turn and talk about what he had done. Next, she had Erica share. Again, the student shared once all the way through and then again more slowly with time for students to restate to their partners. At the end of the discussion, Sussan summarized the mathematical strategies used by students. She asked them to raise their hand to indicate which strategy they used. Sussan would remind them of these strategies the next day as they tackled a similar problem (Figure 1.6).

Figure 1.6 • Sussan listens as her students share their strategies



I noticed that Sussan was engaged in coaching her students not only on the math, but also in developing their own metacognition, including understanding and regulating their emotional response. For example, she listened in on one child, Johan, as he shared his thinking with his partner. As Johan shared, he kept recounting the number of shares, losing track several times of his count. He looked up at Sussan, clearly frustrated, and no longer sure of his answer. Sussan said, “It seems like you are now revising your thinking. What could help you be more sure of your answer?” The student thought and then said, “Recounting by myself.” Sussan nodded, and the student got up and went to the back of the room and began recounting by himself. When he returned to his partner, he was now confident of his answer. He also had practiced noticing his own emotional response and finding a way to become emotionally and mathematically sure of his answer. I saw this kind of strategic coaching multiple times across visits to Sussan’s classroom.

I noticed that some of Sussan's students had different ways of communicating their thinking. Although most students were sitting on the rug during the launch and share, one student, we will call Francisco, was sitting at his own table, a tablet in hand. He appeared to be concentrating on that tablet during the discussion, yet he would occasionally shout out comments that were connected to the problem without raising his hand. Many teachers would have considered his behavior inappropriate. For Sussan, his different way of engaging did not seem to be a problem. She did not call attention to it, but sometimes she ignored his comments and at other times incorporated them into the discussion, restating what he said.

When we debriefed, Francisco's previous special education teacher, who was observing with me, was blown away by this child's engagement, a dramatic improvement from when he was in her class. According to her, Francisco had previously not engaged in any math work that was not one-on-one with a paraprofessional. Francisco solved the problem and was consistently engaged. His engagement just looked different than other kids' engagement. And when seen in the context of his development as an individual, it was something to celebrate indeed! ●

MATHEMATICAL PEDAGOGY OF TRUST

Some fifth-grade teachers would have balked at giving such a challenging problem to their students. In particular, some teachers would worry about giving it to students with disabilities. They might wonder whether the problem somehow had too much language, which would confuse the students. Some teachers would preteach, giving the students a specific procedure to use to solve. Or they would walk them through the problem, step by step. Imagine for a moment how this lesson would have been different if the students were asked to use a procedure for dividing fractions. I remember learning the phrase, "Ours is not to wonder why, just invert and multiply," along with the procedure. In this case, for $\frac{3}{4}$ ths, students would be taught to do the following:

$$12 \div \frac{3}{4} = 12 \times \frac{4}{3} = \frac{48}{3} = 16$$

Does this procedure match how these students were thinking? Forcing students to use this particular procedure, which many students (and adults)

do not understand, would shut down the beautiful proportional and relational thinking that emerged from this problem quite naturally. Sussan believes that sense-making comes first. She trusts in the thinking of her students. She knew they could solve this problem using their mathematical intuition and what they already knew about fractions.

Sussan uses Story Problems as an almost daily routine (Carpenter et al., 2015). She chooses a problem designed to help students explore important grade-level mathematics, as in how to model and understand fraction division. For several days in a row, she gives increasingly complex Story Problems. For more about Story Problems designed to develop deep understanding of fractions, see “Unpacking a Core Idea: Developing Fractional Understanding Through Story Problems” (p. 210).

This routine has a consistent structure that helps students understand and meet expectations for behavior (Table 1.1).

Table 1.1 • Teacher and student actions during Story Problems

Story Problems	Teachers are:	Students are:
Launch	Introducing the story problem in an engaging, comprehensible way using visuals and storytelling.	Listening and restating aspects of the story situation.
Work Time	Conferencing with students as needed to support their thinking, gathering information for the share.	Solving the problems using methods of their choice, talking with peers about their solutions.
Share	Leading a share, ending with consolidation of concepts and/or student strategies.	Sharing (2–3 students) their thinking, either rewriting their own solution or having the teacher do so.

For more about facilitating Story Problems, see the Teaching Practice Guides on CGI Story Problems online, as well as Chapters 8 and 12.



Online Teaching Practice Guide

qrs.ly/l7f7rwq

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

Sussan's classroom is a powerful counterargument to circulating deficit myths about the mathematical potential of students with disabilities. I hear sometimes from educators (and researchers) that students with disabilities cannot "handle" inquiry-based teaching, that these students are not capable of developing their own strategies, and that these students need to be told how to think.

This brings me to perhaps the most important idea in this book. I strongly believe that students with disabilities are underperforming in mathematics in part because we don't trust in their thinking. In fact, we have created *deficit mythologies* about the mathematical capacities of students with disabilities. I write about how these deficit mythologies have emerged and about how we have created systems that perpetuate them. The opposite of deficit thinking is trusting in our students' thinking.

**I strongly believe that students with disabilities
are underperforming in mathematics in part
because we don't trust in their thinking.**

Trust is communicated through the choice of problem, the choice in how to solve it, and the time Sussan gives them to think. Each time I am in Sussan's classroom, I am struck by her silences, her wait time. When she asks a question, she waits. When a student is thinking about what to say, clearly needing an additional minute to formulate their thoughts, she waits. There is no edge to her waiting, no sense that she is anxious to move on. I very much wish that I had this quality as a teacher and a parent. Many of her students have language processing differences, so it might take longer for them to be able to think out a strategy in their mind, or be able to process what she or another student has said. Her waiting—patient, not rushing, kind—was the secret sauce to her students' exceptional engagement in her class.

THIS BOOK

This book offers a vision of how we can design our math classrooms to become spaces in which students with disabilities thrive as mathematical thinkers, spaces that also become more flexible, more supportive, and more welcoming of all of our learners. When we design from the margins, we can transform our classrooms and our schools. A key idea of this book is that what students with disabilities need to succeed in math is not qualitatively different than what all kids need. It is not as if students (without disabilities)

all learn one way, and then students with disabilities all learn another way. Instead, ALL learners are variable. Some students, with and without disabilities, have a particularly hard time paying attention in lectures. That describes a very wide swath of the population, actually. The people with the most challenges in attention might identify as having Attention Deficit Hyperactivity Disorder (ADHD). Since differences in attention occur across everyone, adapting classrooms to work for students with ADHD will not just help those students, but all of us. This is a key idea of Universal Design for Learning (UDL; Meyer et al., 2014), which we explore throughout the book.

A key idea of this book is that what students with disabilities need to succeed in math is not qualitatively different than what all kids need.

The first big idea of the book is that **it matters how we think about disability**. It matters because kids take up how we frame them, and it matters because if educators think students with disabilities cannot do something, they won't give them opportunities to try. In Chapter 2, we will explore different models for understanding disability, such as the medical model, social model, and neurodiversity. We will also explore learner variability, the model for understanding learners used in UDL. Reframing students with disabilities from deficit to asset models is a necessary precondition to trusting their thinking.

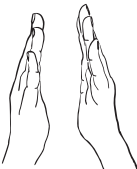
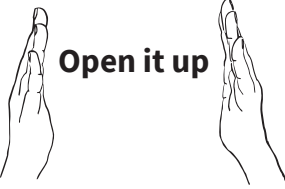
Next we explore how **it matters how we think about the goal of mathematics itself**. Mathematics is not just memorizing. We should not make kids wait to have fun in mathematics until they have endured school mathematics. Math is and can be joyful, creative, and meaningful. Chapter 3 explores these ideas through a study with dyslexic mathematicians.

Deficit thinking about kids leads to deficit pedagogies, which is explored in Chapter 4. This chapter explores the differences between inquiry and explicit instruction, why explicit instruction is recommended so often for students with disabilities, and how to make informed choices about pedagogy. We move beyond seeing these two pedagogies as a binary, and instead we think about how great teachers combine elements of both using a middle school math teacher named Mr. Jay.

The next section of the book provides an in-depth exploration of UDL Math (Lambert, 2021). I have long loved UDL, which is a research-based approach to understanding classrooms and pedagogy grounded in the

learning sciences and neuroscience (Meyer et al., 2014). UDL Math is my version of UDL that is based in research in the learning and teaching of mathematics, as well as in student voice. The big idea in these chapters is that **we can design math classrooms that work for variability by designing from the margins**. Math classrooms focused on meaning-making can be flexible, can be multimodal, and can support strategic development. Chapter 5 is an introduction to UDL Math, including a history of Universal Design. Chapter 6 follows two early childhood teachers, Ms. Diaz and Ms. Murphy, as they redesign group work in Ms. Diaz's inclusive second-grade classroom. They use a simple process that is described in three gestures with my hands (Table 1.2).

Table 1.2 • UDL Math process

 Math class is too narrow	Some math classrooms are narrow, with such rigid ways of engaging that only a few students succeed. The key to understanding barriers is asking students for their input.
 Open it up	After we find barriers, we open up the classroom to allow more students to engage and learn. Opening up a classroom means providing more choice and flexibility, with more agency for students.
 Provide strategic support	Some students will still need more support even after opening up the classroom. Providing strategic support means finding ways to provide support to help students engage and learn.

The final section of the book is a series of classroom stories. We meet teachers as they engage in redesigning some aspect of their practice to better serve disabled and/or neurodiverse students. These chapters are sequenced by the math content the teachers are working on, so that as a reader, you begin with children learning to count and finish with students

learning algebra and fractions. Throughout, we visit classrooms that are inclusive, where the teacher is a general education teacher, as well as special education classrooms, where the teacher is a special education teacher. The teachers are a talented and diverse group, including teachers who identify as neurodiverse or disabled.

In Chapter 7, we explore how to invest time in core ideas. We see the power of repeated engagement in a routine like Counting Collections that helps students develop core ideas about number in a K–1 special education classroom taught by Hannah Benavidez. Special education teacher Kayla Martinez shows how to engage a special education class (K–3) in story problems, while supporting students with language processing challenges (Chapter 8). Dina Williams, teaching an inclusive second-grade classroom, engages in a carefully designed sequence to develop her students’ understanding of the open number line and multidigit addition (Chapter 9).

The next two chapters are about students developing understanding of multiplication and division, and both come from my own research on agentic intervention in multiplicative thinking. In Chapter 10, I explore my experience designing multiplication intervention for two boys in fourth and fifth grade, with a focus on math fact fluency. Chapter 11 tells the story of Yola, an undergraduate researcher who finds a way to increase the participation and the understanding of Inez, a fourth-grade girl, in a number strings routine.

In Chapter 12, we enter Ms. Rey’s inclusive fifth-grade classroom to learn how she creates a classroom environment that supports neurodiverse students, as well as her use of equal sharing fraction problems. In Chapter 13, we follow Kit Golan and their class as they take us through a Connecting Representations Instructional Routine (Kelemanik et al., 2016) in which students connect double number lines with algebraic expressions.

The last two chapters relate to assessment. In Chapter 14, we return to Mr. Jay’s classroom to follow his experiments in assessment and grading in his seventh- and eighth-grade classrooms. Finally, in Chapter 15, we follow special educator Suzanne Huerta as she rethinks Individualized Education Program (IEP) goals in math for her fifth graders.

As you read, you may want to know more about the mathematics content of the chapter. Each classroom chapter (Chapters 7–15) ends with a feature

on the math content. If you want to learn more about a particular teaching practice, such as Which One Doesn't Belong or Story Problems, I have created Teaching Practice Guides on my website (mathematizing4all.com) which have links to resources. Also at my website, you can find my research articles in open access format.



mathematizing4all.com

REFLECTION QUESTIONS

1. What have been your experiences teaching students with disabilities or being a disabled student learning mathematics? What brought you to this book?
2. What did you notice and wonder about Sussan's class doing the 12 chairs problem? What questions would you want to ask her or the students?
3. What do you wonder about doing a story problem in your classroom?
4. What questions do you have about teaching math to disabled students that you hope I answer in this book?