WHAT YOUR COLLEAGUES ARE SAYING . . .

"Too often, in mathematics, we are in a rush to move from the concrete to abstract. In doing so, we overlook the visual. This book is an excellent reminder of the importance of the visual.It not only helps us understand how visuals can help students see mathematics but also helps us see what students see when they are doing mathematics. This book is a necessity for every K–6 teacher."

Peter Liljedahl

Author, Building Thinking Classrooms Burnaby, BC, Canada

"Seeing the Math You Teach delivers exactly what it promises—math you can actually see. The visuals are clean, clear, and do the heavy lifting, making tricky concepts far more intuitive for both students and teachers. If you've ever struggled to explain fractions without resorting to interpretive dance, this book is for you."

Fawn Nguyen

Director, STEM Initiatives Oak View, CA

"If we're serious about equity in math, we must change how we teach it. *Seeing the Math You Teach* gives parents and teachers the tools to do just that—by using visual models and brain-friendly strategies that support all kids, not just those labeled as 'high achievers,' in developing real mathematical understanding."

Pam Seda

Co-author of *Choosing to See: A Framework for Equity in the Math Classroom* Atlanta, GA

"Seeing the Math You Teach is an essential resource for educators looking to deepen their mathematical understanding and better support their students. Through clear visuals and a progression of conceptual development, this book helps teachers see how students think and learn, allowing them to meet each learner where they are. A must-have for building instructional confidence and fostering meaningful mathematical understanding in the classroom!"

Graham Fletcher

Math Specialist Atlanta, GA

"This book is a valuable resource for teachers who want to make math more accessible and meaningful for *all* students. With clear explanations, visual support, and practical strategies, it helps break down abstract concepts so that every learner can engage and build understanding. Grounded in how children learn, this book offers thoughtful guidance for creating lessons that support problem-solving, confidence, and a genuine connection to math."

Melynee Naegele

Instructional Coach, Osage County Interlocal Cooperative Claremore, OK "Teachers often ask, 'How can I ratchet up the quality of my math teaching to better engage my students in mathematical thinking and to deepen their understanding of math beyond answer-getting?' Call this helpful book 'A Compendium of Math Representations' because it provides a slew of topic-specific ways to picture mathematics. By including these multiple representations in our lessons, we ensure that all the active, yet different, brains in our classrooms can 'get it."

Steven Leinwand

Consultant, Mentor, Change Agent Washington, DC

"Seeing the Math You Teach is an essential resource that transforms mathematics into a visual and accessible experience for all students. This comprehensive guide will undoubtedly become a favorite among teachers, showing signs of frequent use as they refer to it time and time again!"

Ann Elise Record

Ann Elise Record Consulting LLC Concord, NH

"Seeing the Math You Teach is a valuable resource that brings clarity to mathematical representations. It will help you make essential concepts more accessible for all learners. The practical strategies and visual models will empower you to foster deeper understanding and greater confidence in math."

John SanGiovanni

Education Coordinator, Instructional Facilitator for Elementary Mathematics, Howard County Public Schools Westminster, MD

"I've shared about the 'Sweet Spot of the C-R-A model,' where you do an activity that includes all 3 and the power that has with building your students' understanding. *Seeing the Math You Teach* is full of representations to help you get to that 'Sweet Spot.'"

Christina Tondevold

Founder,Build Math Minds Orofino, ID

"If you want to see math visually, then this is the book for you! *Seeing the Math You Teach* shows us what math looks like. It's an easy-to-read reference book that will be on my desk to share with teachers (and students!)"

Laura Vizdos Tomas

K–5 Math Coach, School District of Palm Beach County Cofounder,LearningThroughMath.com West Palm Beach, FL "With clear explanations and powerful visual representations, *Seeing the Math You Teach* is an indispensable resource for K–6 teachers looking to deepen their understanding of mathematics and how students learn it. Packed with practical strategies, this book empowers educators to guide students beyond rote memorization toward true mathematical comprehension by bridging the gap between abstract concepts and hands-on, visual learning. A must-have for any teacher committed to helping students see, explore, and master math with confidence."

Chase Orton

Author, The Imperfect and Unfinished Math Teacher Culver City, CA

"Seeing the Math You Teach is an essential resource for teachers, instructional aides, special educators, and those entering the profession through alternative certification. As a district math coordinator, I appreciate how the book's clear visuals, concise text, and embedded video clips make complex mathematical representations immediately accessible—not just for educators, but also for parents and caregivers supporting math learning at home. An eye-opening approach to visualizing mathematics!"

Tara Fulton

K–8 Math Coordinator, Crane School District Yuma, AZ

"This book is a must-read for educators and caregivers who want to foster sense-making in mathematics. The authors present clear, comprehensive visual representations that make abstract concepts understandable. A valuable resource for anyone who is eager to provide children with rich and connected mathematical experiences."

Amy Chang

Official Team Member, Building Thinking Classrooms South Hadley, MA

"Seeing the Math You Teach is more than just a resource—it's a toolbox for teachers, packed with practical ideas to help make math visible and meaningful. It gives you simple ways to break down big concepts and help every student truly get it."

Bryan Borden

Instructional Coach, Buckeye Elementary School District Buckeye, AZ

SEEING Corrections MATH YOU TEACH



KIMBERLY RIMBEY KATIE BASHAM CHRYSTE BERDA



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Note From the Publisher: The authors have provided video and web content throughout the book that is available to you through QR (quick response) codes. To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand.

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From Kimberly Ann Rimbey:

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Today, I am thrilled to bring you this new version, made possible by the outstanding partnership with the Corwin team. Once again working alongside Katie and Chryste has been an honor, and I am so excited to share this resource with you, your students, and their families.

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Math makes sense. I wish that I'd known this more finitely when I was a young, impressionable student. I wish my teachers had known it. Thank you to my parents, who never questioned my capabilities and who, decades later, continue to have confidence that I am capable of anything.

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xv

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Georgina Rivera Principal, Charter Oak International Academy Hartford, CT

About the Authors



Kimberly Rimbey is an author, inventor, entrepreneur, speaker, consultant, coach, mentor, advocate, and, proudest of all, teacher through and through. Kim currently serves as the Chief Learning Officer and CEO at KP® Mathematics and an official Building Thinking Classrooms consultant alongside Peter Liljedahl. A lifelong teacher and learner, her heart's work centers on equipping teachers and helping them fall in love with teaching and learning over and over again.

Always a teacher at heart, Kim has held several leadership positions, including

Executive Director of Curriculum and Instruction, Chief Learning Officer, and Mathematica Program Area Coordinator. That said, everything Kim has done in her career is based on what she learned during her eighteen years as a mathematics coach and classroom teacher.

Kim is National Board Certified in Early Adolescent Mathematics, and she is a recipient of the Presidential Award for Excellence in Mathematics Teaching. Kim is the co-inventor of KP® Ten-Frame Tiles and has authored and co-authored several publications, including *Mastering Math Manipulatives* and *Meaningful Small Groups in Math* for Corwin, *Math Power: Simple Solutions for Mastering Math* for the Rodel Foundation of Arizona, and, most recently, *The Amazing Ten Frame* series for KP Mathematics.

Kim earned her bachelor's degree in Elementary Education and Mathematics from Grand Canyon University, her master's degrees in Early Childhood Education and Educational Leadership from Arizona State University and Northern Arizona University, and her PhD in Curriculum and Instruction from Arizona State University. Kim lives in Phoenix, Arizona, where she continues to inspire teachers and their leaders.

xvii



Katie Basham is an aggressive Scrabble player, avid reader, and an affectionate mother and wife. She is also an instructional specialist in the Pacific Northwest, where she leads professional development and works to advance the learning of teachers both in the classroom and in collaborative groups. She is a former classroom teacher, elementary math specialist, and assessment coordinator. Katie strives to support teachers and teacher-leaders in their efforts to improve mathematics learning opportunities for all students. She is active in state and national

mathematics organizations and has served on the Arizona Math Leaders Board of Directors. She received her bachelor's degree from Boston College and her master's degree from Northern Arizona University. Katie is a self-proclaimed "nerd" and proud of it; in addition to her love of numbers she is also an avid reader and enjoys walks on the beach with Honey, the family's Silver Labrador.



Chryste Berda is an adventurer, traveler, teacher, coach, consultant, author, and mom of two amazing Gen Z guys. She finds joy in singing at the top of her lungs and in the art of teaching. Chryste is energized by sharing her passion for learning with her colleagues as the district math coordinator and as a Regional VP for the Arizona Association of Teachers of Mathematics. She is intensely curious about students' thinking and spends much of her time listening to students explain their ideas. Chryste is an Honors program graduate from Western Oregon University with

a bachelor's degree in Elementary Education and Interdisciplinary Studies with a specialty in Mathematics. She holds two master's degrees from Arizona State University—one in Curriculum and Instruction, the other in Educational Leadership. She has taught mostly math to students in Grades K–12, their teachers, and leaders (in addition to some other very fun courses) since 1998. Chryste is a native Oregonian who for the past twenty-ish years has lived with her family in Arizona's Valley of the Sun. She and her husband are raising two Arizona natives and aspire to become "snowbirds" in the future.

Introduction

Hello, friend!

Spoiler alert—everyone can be good at math.

Read that again. **Everyone** can be good at math. This is true for each of the students in your classroom, and it is true for **you** as well.

Yes, daily math instruction may look different and feel different than it did when you were a student. Maybe you loved math as a child; maybe you struggled. Either way, you recognize that the daily math instruction you have been charged with teaching involves understanding, flexibility, and the ability to explain and visualize abstract thinking.

The teacher's role has changed. No longer are we disseminators of information, holders of "all the knowledge" that we bestow to the students in our charge. Instead, we are moderators of learning. We guide student thinking to deeper levels of understanding by providing manipulatives, crafting questions, and guiding student inquiry.

What a wonderful opportunity we have been given! As teachers we have the unique privilege to foster a child's love of mathematics, of numbers, and of learning! And again—**everyone** can be good at math.

What an immense responsibility this is. Relying on the manner in which you were taught and your feelings about math (positive or negative) will no longer serve the needs of the students you teach. You are looking for resources and support to increase your own sense of math efficacy. Your desire is to increase your "toolbox."

Congratulations . . . this is the book you've been looking for!

So, who are we? We are coaches, mentors, authors, and above all else, teachers. For a collective of eighty-five-plus years, we have embraced the magic of making math come alive for children and their teachers, and we are thrilled to share this book with you. We believe deep-down that math makes sense. We believe that everyone can be good at math.

Being good at math is about so much more than memorizing equations. Thank you for accepting the challenge to see the math you teach differently. Thank you for seeking clarity. Most importantly, thank you for wanting what is best for all of the children you serve.

We're in this together!

1

Kimberly, Katie, and Chryste

What This Book Is About

This book is intended to help **you** help your children. It was created to provide you with images and descriptions of the way our students learn and think. The images focus on students' understanding of how and why math works, and they are explained in plain and simple terms.

Why This Matters

The strategies and operations that pop into your adult mind when you think "math" are likely abstract. They may involve symbols and procedures that were "gifted" to you by a caring adult, one who showed you a series of steps that you were then asked to memorize and apply. What we know about math today includes focused attention on developing a deep and sustainable understanding of problems, as well as the perseverance to solve them. Furthermore, understanding and perseverance must be intentionally developed by building on and making connections within the learner's own understanding.

How Mathematics Learning Happens

Deep understanding in mathematics evolves naturally when students use physical (concrete) materials such as counters, linking cubes, or even cars pulled from a toy box or fruit from a bowl. In this "doing" stage, learners are modeling strategies and operations with physical manipulation.

Similarly, learners can also use visual (pictorial) representations to help them "see" the math. This is when they create visuals that either represent concrete objects such as pictures or sketches, or they construct more abstract representations such as number lines or number bonds to explain their thinking. At first, student-created drawings are littered with detail. For example, when making sense of a problem such as *There were 10 bunnies on the hill, 3 bunnies hopped away...* the learner may draw complete pictures of bunnies with tails, big feet, and floppy ears. With the right prompting and support, students in this stage can understand that a circle, even one drawn in a tens frame, can represent a bunny. An important part of this mode centers on students making connections between the physical and visual representations.

By making connections between the physical and visual representations, students provide meaning that can also be connected to symbolic understanding, sometimes referred to as the "abstract mode." Here, learners use numbers and other mathematical symbols to model their thinking. A studio artist we know well once said when talking about human anatomy, "People think that they can make abstract art—they think it's easy to just throw paint and shapes on a canvas. A real artist, however, must understand the foundation: the location of the bones, the shape of the muscles. Only when you know how the form looks at its foundation can you make an abstraction of something such as the human form."

In this abstract form, learners can think critically about how they might represent math and their own understanding by basing it on and making connections between the different representations. Note that as students make connections between and among physical, visual, and symbolic representations, their understanding deepens even further when they talk about their thinking (verbal representations) and connect their thinking to real life (contextual representations). It's the connections between each representation that lead to deeper understanding.

Figure i.1 shows an example of how students might connect representations for place value using base-ten blocks, sketches, and symbols.

 Figure i.1
 An Example of Concrete, Representational, and Abstract Representations of Place Value



Figure i.2 shows an example for how learners might connect representations for equivalent fractions using two-color counters, sketches, and symbols.



Figure i.2 Concrete, Representational, and Abstract Representations of Equivalent Fractions

Figure i.3 show how students might connect representations for adding integers using algebra tiles, sketches, and symbols.

Figure i.3 Concrete, Representational, and Abstract Representations of Adding Integers



Introduction

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Unfortunately, our school system is often steeped in doing things as they've always been done. This is especially true in math class. While there has been great effort in the last fifteen to twenty years to focus on improving conceptual understanding, there are still many classrooms in which students (and sometimes teachers) often rush to abstract and procedural mathematics, or "just numbers," without spending the time needed to develop deeper understanding with objects, tools, drawings, and the like. When this occurs, math learning is short-circuited as we resort to memorization rather than deep understanding.

Part of your role as a math teacher centers on developing your own understanding of the many ways both to see and represent the math you teach. By knowing and understanding these strategies, you are able to support and encourage students' thinking as they select and create their **own** representations. Only then do students make sense of mathematics; it is the **students** who should be choosing and creating the physical and visual representations, not simply mimicking what the teacher shows them. Please do your best to avoid "show-and-tell teaching"; instead, support students in the ways **they** choose how to show their thinking.

What the Role of Teacher Looks Like

A skillful teacher is able to probe for understanding by going back and forth between the physical, visual, and symbolic representations, using conversation to help the learner verbally craft connections between their thinking and the way they choose to represent it, and then connect those understandings to contexts in their own lives.

If you are most familiar and accustomed to one or maybe two particular algorithms or strategies for any given operation, this is the perfect opportunity for you to make your own connections. In the pages that follow you will find not only drawings and images of student thinking, but links to videos that model the use of manipulatives. We urge you to ask yourself, "How does this fit with what I know?" "How does this strategy mirror the strategies that I am familiar with using?" "What new ideas does this give me?" We are here to help you answer all of these questions.

Why These Representations?

It is important to note that we have chosen to model many of the most common strategies developed by learners, those that we see most often in the classroom. Kids, however, are always full of surprises. Some will be likely to invent their own strategies, which may be given equal consideration. Exploration and discussion of student strategies should not be seen as an opportunity for the teacher to **show** students what to "do" and ask that they replicate it. When we acknowledge student thinking and sense-making as a whole group by sharing strategies and asking thoughtful, probing questions, we empower all students. And that is our most basic intent. We wish to empower you with additional knowledge and tools to enhance your own clarity and understanding in mathematics. We believe you will find what you're looking for here.

How to Use This Book

This is not the kind of book you read cover-to-cover. You will most likely use this book as a reference, jumping around as needed to inform the math teaching and learning going on in your classroom. We anticipated you would want to jump around, and we organized the book accordingly.

Let's take a minute to show you how this guide is put together.

There are sixteen color-coded chapters that can be used to quickly locate a topic, and you can jump around to find just what you need to know. Chapters begin with basic topics and go into more detail as you move toward the end. On pages ix-xiii, you'll see a list with all of the different math topics covered in this book. Look for the matching color on the edge of the page to go directly to the chapter.

There is also an additional tool at the back of the book to help you locate specific math problems: the **Topic Index**.

The **Topic Index** lists common math words alphabetically. This more typical way of listing common topics addressed throughout the book provides a handy way to look up topics such as "unit fractions" or "number lines."

Over the years, many teachers have found a variety of ways to use this book in supporting math teaching and learning. Some of those include using the book the following ways:

- As a preparation guide for self-learning about multiple ways to represent mathematical ideas.
- As a planning resource to anticipate a variety of student representations ahead of time.
- During PLC discussions with colleagues.
- To raise family awareness of how children may represent math when doing homework—often by copying a page or two and sending it home ahead of time.
- To support student awareness of representations they may choose from again, often by copying (or projecting) a page or two and sharing them with students.
- As a reference to the math taught in your curriculum resources or textbooks to provide additional support for representing math thinking, especially in places where the textbook is heavy with "numbers only" work that would benefit from more student visuals.

These are just a few samples of how this book can be used. We encourage you to make it your own, finding new and innovative ways to support your own learning as well as that of your students and their families.

You will note that there are also several videos sprinkled throughout the volume. Since this book primarily centers on visual representations, we wanted to provide support for connecting visuals to physical objects (e.g., manipulatives). Therefore, these videos provide brief snippets for how you might help students connect physical, visual, and symbolic representations. As mentioned before, it's important that you use these to guide your thinking as you design instruction that facilitates students' thinking rather than resorting to "show-and-tell teaching."

We hope you find this to be a valuable resource to validate, reinforce, and expand your thinking and understanding... truly being able to "see" the math you teach so that your students can see it too.

Chapter 1

Whole Number and Decimal Place Value

Students come to school with a natural sense of quantity. They are born hardwired to compare quantities and determine which has more or which has less. That said, most students typically require direct opportunities with increasingly complex ideas about number and quantity to gain a greater understanding of our base-ten number system.

During the primary grades, the foundation for the base-ten system is built by developing a focused understanding of place value. This includes understanding that the entire base-ten system is made up of only ten digits (0–9). Once we have counted 0–9 to reach ten, we have a new unit made from a group of ten ones, called "a ten" for short. We represent this new unit by writing two digits, 1 and 0, side-by-side. This first grouping of ten may seem simple, but it's quite a complex idea for a young child.

As numbers increase in size and value, we record digits side-by-side to represent the number of groups in each "place." The ones place

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shows 0–9 units, the tens place shows how many groups of 10, the hundreds place shows how many groups of 100, and so on. Each group of three digits (ones, tens, and hundreds) is called a "period," and this pattern is repeated over and over to help us understand and read numbers. Each period includes a three-digit number, separated from other periods with a comma, to represent thousands, millions, billions, etc., that efficiently represents larger numbers.

Students develop an understanding that as you move left from digit to digit, the "magnitude" of each place is always ten times greater than the place to its right; and similarly, the magnitude of the place to the right is always one-tenth the value to its left. This sets them up to understand powers of ten relationships and decimals less than one. Without these understandings, students may struggle to develop more sophisticated ways of counting and comparing quantities as well as work more complex math operations.

Typical Trajectory in Most State Standards Frameworks:

- K-1: Whole numbers within 20
- Grades 1-2: Whole numbers within 100
- Grades 2-3: Whole numbers within 1,000
- Grades 4–5: Whole numbers within 1,000,000 and decimals through the hundredths places
- Grades 5–6: All whole numbers and decimals through the thousandths place

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Ones and Tens on a Ten Frame

A ten frame is an array (group) of squares that can be easily used to visualize numbers between 0 and 10, which helps with visualizing how numbers can be combined or broken apart.



Unitizing is understanding that the number "ten" can be thought of as both 1 group of ten and 10 individual ones. 34 10 °00 l see l see ten + 24 ones. 34 ones 10 10 10 10 10 °00 °00 l see l see 2 tens + 14 ones. 3 tens + 4 ones.

Place Value

Whole Number Place Value

Multi-digit numbers can be understood by looking closely at the **place value** of each digit.



Whole Number and Decimal Place Value

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Whole Number Place Value (continued).....

Video 1: Place Value Pieces: You can use base-ten blocks, PV chips, and a variety of other place-value pieces to represent whole numbers, decimal fractions, and operations. Pictured here: base-ten blocks, place-value disks, and KP[®] Ten-Frame Tiles

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Whole Number Place Value (continued)



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Place Value Organization

The base-ten system is made up of only ten digits (0-9). Once we have counted 0–9 and reached ten, we have a new unit made from a group of ten ones, called "a ten" for short.



What happens in the ones place, happens in every place. So when we've counted 9 tens and reached 10 tons we have group of 10 tens, called "a hundred" for short.



Place Value Organization (continued)

We represent this new unit by writing three digits, 1, 0, and 0, side-by-side to record the number of hundreds (1), tens (0), and ones (0).

Hundreds	Tens	Ones			
1	0	0			

The same is true for the periods. What happens in the ones period happens in every period. A **period** is a set of three digits representing groups of ones, tens, and hundreds of units that is repeated over and over with different magnitudes.

Millions Period				Tł	nousar Period	nds 1		Ones Period		
Hundreds	Tens	Ones		Hundreds	Tens	Ones		Hundreds	Tens	Ones
		1	9	0	4	1	9	7	1	٩

Whole Number and Decimal Place Value

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Decimal Place Value

You can use a **decimal** to represent a fractional part of a number that is less than one whole. Decimals are commonly used to represent money.



Source: Bill image by istock.com/SereiKorolko; Coin images by istock.com/filo



The 10-to-1 relationship described with money works for all digits that are side-by-side in decimal place value as you move from right to left.



Decimal Place Value (continued)

A decimal number can be understood by looking closely at the **place value** of each digit and seeing that it takes ten of the value to the right to make one of the value immediately to the left. (You can use a **decimal** to represent the fractional part of a number that is less than one whole.)



Whole Number and Decimal Place Value

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Decimal Point.....

Just as there is space between the whole number part and the fractional part of a number like $3\frac{12}{100}$, we use a **decimal point** to create space between the whole number and the fractional part of a number.



As digits are added to the left of the decimal point, we are representing counting more "things." As digits are added to the right of the decimal point, we are no longer counting more "things"; instead, the digits

represent a single fraction that is less than one, and we are simply increasing the precision of that fraction.

Whole Number Part										Frac les	tion that	Part In 1		
M F	Millions Period			Thousands Period			ones Period			Deu	Decimal Period		al d	
Hundreds	Tens	Ones		Hundreds	Tens	Ones		Hundreds	Tens	Ones	simal Point	Tenths -12	Hundredths - 10	Thousandths - 100
		1	9	0	4	1	9	7	1	9	•	3	4	5

Comparing Numbers.....

Both large and small numbers can be compared using place value. The symbols >, =, and < are used to depict the comparisons (see page 22 for more about math symbols).







Place Value

Whole Number and Decimal Place Value

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Comparing Decimals

Because the number system is uniform, students can use the same place value understanding for comparing larger numbers and smaller numbers.





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