WHAT YOUR COLLEAGUES ARE SAYING . . .

"This text provides a clear definition of what fluency really is and provides strategies to deepen students' number sense and make them fluent mathematicians."

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Third-Grade Teacher

"I wish I'd had this book when I was in the classroom 10 years ago. The authors clearly lay out a pathway to procedural fluency, including intentional activities to understand and practice specific strategies, while also advocating for space for students to make decisions and feel empowered as mathematical thinkers and doers."

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Instructional Designer
CenterPoint Education

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District 93

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Instructional Coach
Upper Grand District School Board

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Nicole Rigelman
Professor of Mathematics Education
Portland State University

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"As principal of a Title I school and former mathematics specialist, I appreciate this resource for what it is; a true teacher's companion! The authors provide explicit strategy instruction in whole number multiplication and division to foster flexibility, efficiency, and true fluency in ALL students."

Allie S. Watkins
Principal
Frederick County Public Schools

"This is the book classroom teachers have been waiting for! It provides extensive support as students gain confidence with multiplying and dividing whole numbers, and it does so with a deep esteem for educators and learners. The authors, highly respected in the field of mathematics education, provide careful explanations of effective strategies that will help students build their number sense and their computational fluency. There are routines for supporting instruction in the classroom, games and centers for practicing the strategies, and prompts to encourage sense making and extend learning. Kudos to all three authors for writing such an important, no-nonsense book. I cannot wait to share it with my colleagues!"

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Grades 4 & 5 Spanish Bilingual Teacher
Calgary Board of Education

"For years research has indicated that fluency is much more than speed, yet timed assessments and traditional instruction persist for teachers without a clear vision or tools to change their practices. This series provides teachers with the explicit examples, resources, and activities needed to bring that research to life for their students and will quickly become a well-worn guidebook for every fluency-focused classroom. This is the toolkit teachers have been yearning for in their journey toward fluency with their students."

Gina Kilday
Elementary Math Interventionist and MTSS Coordinator

"Fluency isn't a dry landscape of disconnected facts—it is a rich soil for developing and connecting diverse perspectives and ideas. This book series equips you with a deep understanding of fluency and a variety of activities to engage students in co-constructing ideas about multiplication and division that will last a lifetime."

Berkeley Everett
Math Coach and Facilitator for UCLA Mathematics Project
Math Consultant for DragonBox

"This is the book we (educators) have been waiting for! *Figuring Out Fluency – Multiplication and Division With Whole Numbers* provides classroom teachers, special ed teachers, and instructional coaches with the support they need to develop true fluency for every child. This book is packed with great activities; quality practice that is not a worksheet; examples, routines, games, and centers; and a companion website with resources ready to use! Any question you may have—about how and when to teach a specific strategy, how to engage your students in meaningful practice, or assessing your students' fluency—will be answered in this book! Any educator that is serious about understanding how basic fact

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strategies grow into general reasoning strategies and how to advance their students' fluency will want this book! True fluency is a way of thinking, rather than a way of doing!"

Melisa Jean Hancock
Mathematics Consultant

"*Figuring Out Fluency – Multiplication and Division With Whole Numbers* is a must-have for all educators who teach multi-digit multiplication and division. With a focus on equity embedded throughout, this practical, ready-to-use resource provides everything needed to teach students to become strategic thinkers while giving *all* children access to reasoning."

Nichole DeMotte
Atkinson Academy
K–5 Mathematics Coach

"This book—indeed this *series*—is a must-read for elementary and middle level teachers, coaches, and administrators. Within this resource you will find a synthesis of important research organized to help readers develop a clear and common understanding of fluency paired with a large collection of teaching activities that provide concrete ways to support students' fluency development. *Figuring Out Fluency* provides a much-needed roadmap for teachers looking to increase computational proficiency with multiplication and division."

Delise Andrews
3–5 Mathematics Coordinator
Lincoln Public Schools

"The authors John J. SanGiovanni, Jennifer M. Bay-Williams, and Rosalba Serrano shine a bright light on how math fluency is *the* equity issue in mathematics education. How refreshing to have a book that equips math educators with the research and strategies to make a difference for *all* students! Let's implement these strategy modules in this book and help kids figure out fluency once and for all!"

Kelly DeLong
Executive Director for the Kentucky Center for Mathematics
Northern Kentucky University

"In *Figuring Out Fluency – Multiplication and Division With Whole Numbers*, John J. SanGiovanni, Jennifer M. Bay-Williams, and Rosalba Serrano have provided readers with a thorough education that guides them through the entire fluency journey. Grounded in research and packed with illustrative examples, this book is a must-have that delivers practical strategies, tools, resources, and recommendations that will immediately enhance your practice. In a sea of books on fluency, this one stands out. In a word . . . wow!"

Alison J. Mello
Assistant Superintendent Foxborough Public Schools
Author
Math Consultant

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"John J. SanGiovanni, Jennifer M. Bay-Williams, and Rosalba Serrano hit the mark with *Figuring Out Fluency – Multiplication and Division With Whole Numbers*. The activities are fun and engaging and encourage students to think more flexibly and fluently as they work with multiplication and division strategies. Bravo!"

Joshua Barnes
Third-Grade Teacher

"The number one topic teachers grapple with every year is fluency. This companion book is an all-encompassing resource that takes the guess work out of multiplication and division fluency instruction and practice! With a focus on reasoning, understanding, and reasonableness, this is a practical, easy-to-follow guide for teachers. The strategy modules contain teaching prompts, routines, games, center activities, and a very critical component missing in most resources—strategy briefs for families."

Marissa Walsh
Elementary Math Instructional Coach
Blue Springs School District

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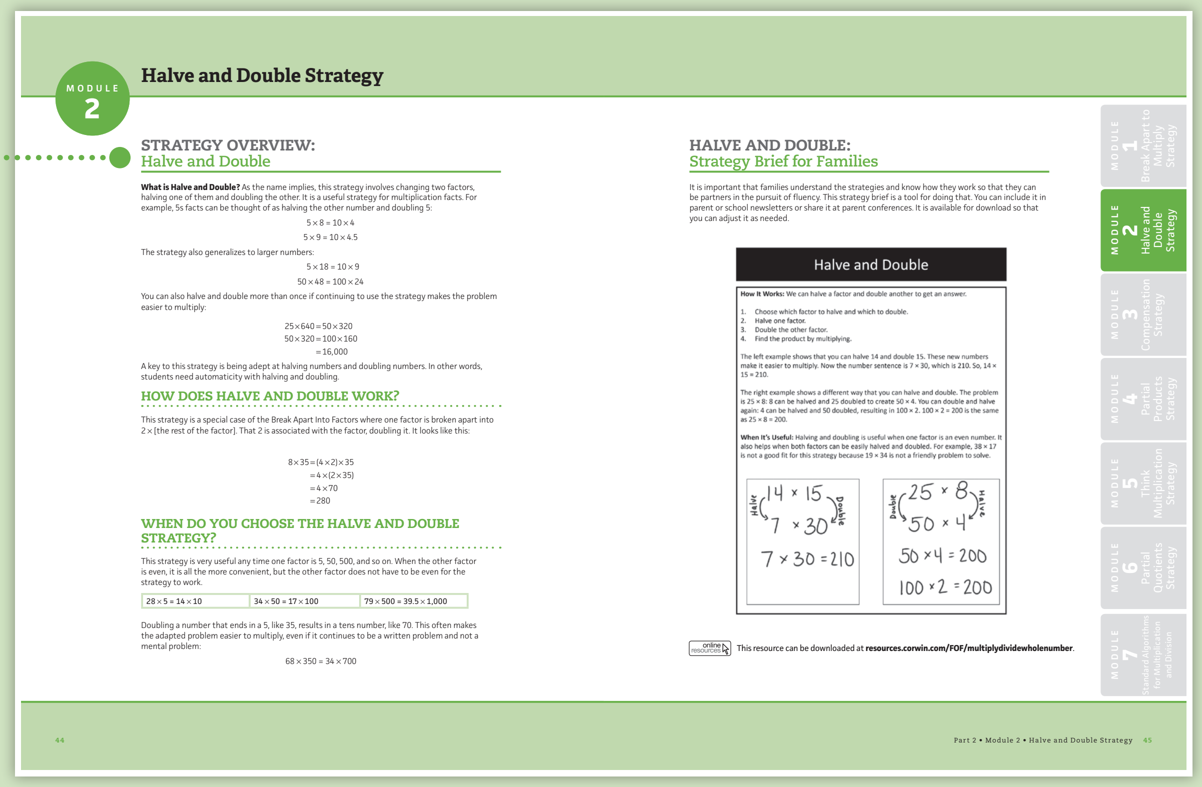
Figuring Out Fluency—Multiplication and Division With Whole Numbers: A Classroom Companion
The Book at a Glance

Building off of *Figuring Out Fluency*, this classroom companion dives deep into five of the Seven Significant Strategies, plus the standard algorithms, that relate to procedural fluency when multiplying and dividing whole numbers, beyond basic facts.

FIGURE 13 ● Reasoning Strategies for Multiplying and Dividing Whole Numbers

REASONING STRATEGIES	RELEVANT OPERATIONS
1. Break Apart to Multiply (Module 1)	Multiplication
2. Halve and Double (Module 2)	Multiplication
3. Compensation (Module 3)	Multiplication
4. Partial Products and Quotients (Modules 4 and 6)	Multiplication and Division
5. Think Multiplication (Module 5)	Division

Strategy overviews and family briefs communicate how each strategy helps students develop flexibility, efficiency, accuracy, automaticity, and reasonableness.



MODULE 2

Halve and Double Strategy

STRATEGY OVERVIEW: Halve and Double

What is Halve and Double? As the name implies, this strategy involves changing two factors, halving one of them and doubling the other. It is a useful strategy for multiplication facts. For example, 5s facts can be thought of as halving the other number and doubling 5:

$$5 \times 8 = 10 \times 4$$
$$5 \times 9 = 10 \times 4.5$$

The strategy also generalizes to larger numbers:

$$5 \times 18 = 10 \times 9$$
$$50 \times 48 = 100 \times 24$$

You can also halve and double more than once if continuing to use the strategy makes the problem easier to multiply:

$$25 \times 640 = 50 \times 320$$
$$50 \times 320 = 100 \times 160 = 16{,}000$$

A key to this strategy is being adept at halving numbers and doubling numbers. In other words, students need automaticity with halving and doubling.

HOW DOES HALVE AND DOUBLE WORK?

This strategy is a special case of the Break Apart Into Factors where one factor is broken apart into 2 × [the rest of the factor]. That 2 is associated with the factor, doubling it. It looks like this:

$$8 \times 35 = (4 \times 2) \times 35 = 4 \times (2 \times 35) = 4 \times 70 = 280$$

WHEN DO YOU CHOOSE THE HALVE AND DOUBLE STRATEGY?

This strategy is very useful any time one factor is 5, 50, 500, and so on. When the other factor is even, it is all the more convenient, but the other factor does not have to be even for the strategy to work.

28 × 5 = 14 × 10	34 × 50 = 17 × 100	79 × 500 = 39.5 × 1,000

Doubling a number that ends in a 5, like 35, results in a tens number, like 70. This often makes the adapted problem easier to multiply, even if it continues to be a written problem and not a mental problem:

$$68 \times 350 = 34 \times 700$$

HALVE AND DOUBLE: Strategy Brief for Families

It is important that families understand the strategies and know how they work so that they can be partners in the pursuit of fluency. This strategy brief is a tool for doing that. You can include it in parent or school newsletters or share it at parent conferences. It is available for download so that you can adjust it as needed.

Halve and Double

How It Works: We can halve a factor and double another to get an answer.

1. Choose which factor to halve and which to double.
2. Halve one factor.
3. Double the other factor.
4. Find the product by multiplying.

The left example shows that you can halve 14 and double 15. These new numbers make it easier to multiply. Now the number sentence is 7 × 30, which is 210. So, 14 × 15 = 210.

The right example shows a different way that you can halve and double. The problem is 25 × 8: 8 can be halved and 25 doubled to create 50 × 4. You can double and halve again: 4 can be halved and 50 doubled, resulting in 100 × 2. 100 × 2 = 200 is the same as 25 × 8 = 200.

When It's Useful: Halving and doubling is useful when one factor is an even number. It also helps when both factors can be easily halved and doubled. For example, 38 × 17 is not a good fit for this strategy because 19 × 34 is not a friendly problem to solve.

online resources This resource can be downloaded at **resources.corwin.com/FOF/multiplydividewholenumber**.

MODULE 1 Break Apart to Multiply Strategy
MODULE 2 Halve and Double Strategy
MODULE 3 Compensation Strategy
MODULE 4 Partial Products Strategy
MODULE 5 Think Multiplication Strategy
MODULE 6 Partial Quotients Strategy
MODULE 7 Standard Algorithms for Multiplication and Division

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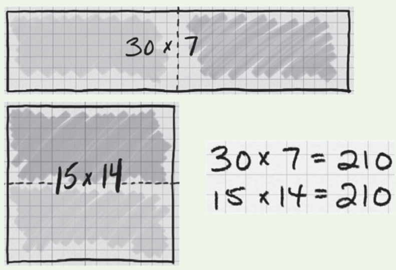
Each strategy module starts with teaching activities that help you explicitly teach the strategy.

TEACHING ACTIVITIES for Halve and Double

Before students are able to choose strategies, a key to fluency, they first must be able to understand and use relevant strategies. These activities focus specifically on Halve and Double. Halve and Double is a strategy that works well in certain cases. While students may employ other methods, which is appropriate, they also must learn this strategy to develop their flexibility and efficiency. Students must understand that halving a factor while doubling the other yields the same result. You can develop this concept by working with a variety of representations and connecting them to equations. In this section, we focus on instructional activities for helping students develop understanding of the Halve and Double strategy and for determining when it is appropriate. Fluency with this strategy will build as students develop skill with halving or doubling numbers in general.

ACTIVITY 2.1
HALVING AND REARRANGING RECTANGLES

In this activity, you pose rectangles to students for them to trace on grid paper and cut out. Rectangle dimensions to use could include 15×14, 28×5, 16×6, 25×22, and 16×16. Students record the equation for the area of one of their rectangles. Then, they cut the rectangle in half and rearrange it to make one, longer rectangle and record the equation for it. In the following figure, the original rectangle was 15×14. The student cut it in half, creating two rectangles that were each 15×7. Then, she aligned those two smaller rectangles, creating a rectangle that was 30×7. She recorded the equations for each rectangle's area.



Students repeat halving and rearranging for the other rectangles that you pose. As students work, you want them to think about how the factors and the area of the corresponding rectangles are similar and different. After students complete the activity, a group discussion should be had about how one factor is halved and how the other is doubled yet the product (area) remains the same. Ask students to determine if this idea always works and then have them find examples to prove their case. It's possible that students will overlook that one factor is always even. If so, be sure to insert a rectangle, like 13×15 or 17×21, for them to explore.

Even though this is an introduction to the concept of halving and doubling, it isn't too soon to discuss when this technique might be a good idea. Notice that 15×14 and 28×5 from the suggested set create expressions that yield multiples of 10 (30×7 and 14×10, respectively). Because of this, the Halve and Double strategy is a good choice for these expressions.

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Each strategy shares worked examples for you to work through with your students as they develop their procedural fluency.

WORKED EXAMPLES

Worked examples are problems that have been solved. Correctly worked examples can help students make sense of a strategy and incorrectly worked examples attend to common errors.

Halve and Double is useful in special cases, so a major challenge is to just remember it *is* a strategy and be on the lookout for when it will work. Other challenges include the following:

1. The student makes an error in halving an odd number.
   - 65×5: changes to 38.5×10.
2. The student has trouble choosing which number to halve and which to double.
   - 450×12: changes to 225×24, rather than 900×6.

A key idea is to look for a number that, when doubled, is more manageable to multiply. In addition, noticing a second opportunity to use Halve and Double can continue to make the problem easier to work. Worked examples can attend to these ideas. The prompts from Activity 2.5 can be used for collecting worked examples. Throughout the module are various worked examples that you can use as fictional worked examples. A sampling of additional ideas is provided in the table.

SAMPLE WORKED EXAMPLES FOR HALVE AND DOUBLE

Correctly Worked Example (make sense of the strategy) What did ______ do? Why does it work? Is this a good method for this problem?	Trevor's work for 250×28: 250×28 500×14 $1{,}000 \times 7 = 7{,}000$
Partially Worked Example (implement the strategy accurately) Why did ______ start the problem this way? What does ______ need to do to finish the problem?	Neesha's start for 15×260: $30 \times 130 =$ Dante's start for 25×68: $50 \times 34 =$
Incorrectly Worked Example (highlight common errors) What did ______ do? What mistake does ______ make? How can this mistake be fixed?	Greta's work for 50×45: $100 \times 23.5 = 2{,}350$

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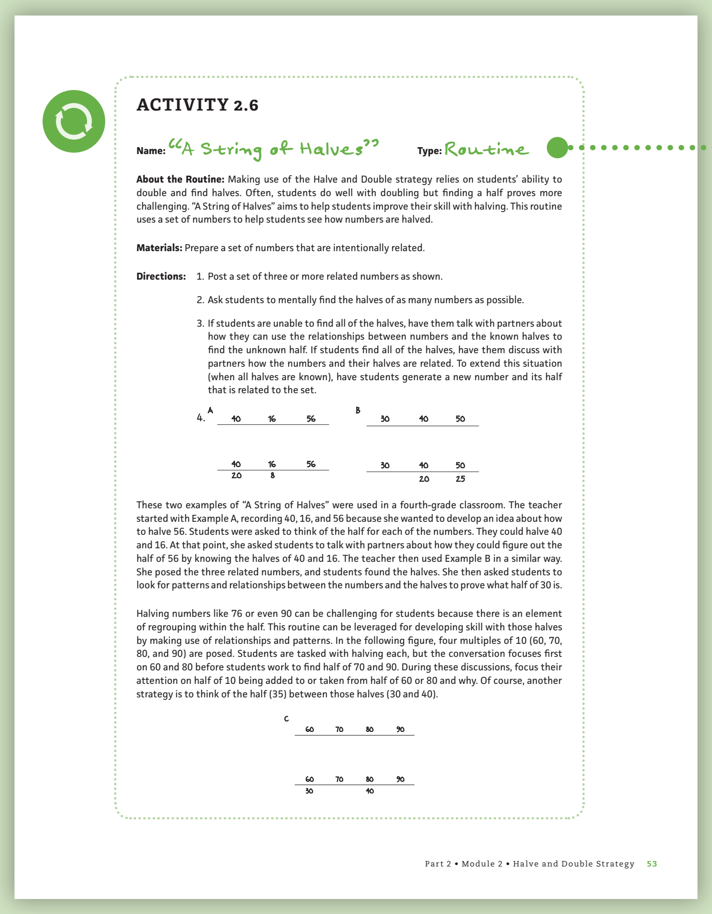

ACTIVITY 2.6

Name: "A String of Halves" **Type:** Routine

About the Routine: Making use of the Halve and Double strategy relies on students' ability to double and find halves. Often, students do well with doubling but finding a half proves more challenging. "A String of Halves" aims to help students improve their skill with halving. This routine uses a set of numbers to help students see how numbers are halved.

Materials: Prepare a set of numbers that are intentionally related.

Directions:

1. Post a set of three or more related numbers as shown.
2. Ask students to mentally find the halves of as many numbers as possible.
3. If students are unable to find all of the halves, have them talk with partners about how they can use the relationships between numbers and the known halves to find the unknown half. If students find all of the halves, have them discuss with partners how the numbers and their halves are related. To extend this situation (when all halves are known), have students generate a new number and its half that is related to the set.
4. A

40	16	56

40	16	56
20	8	

B

30	40	50

30	40	50
	20	25

These two examples of "A String of Halves" were used in a fourth-grade classroom. The teacher started with Example A, recording 40, 16, and 56 because she wanted to develop an idea about how to halve 56. Students were asked to think of the half for each of the numbers. They could halve 40 and 16. At that point, she asked students to talk with partners about how they could figure out the half of 56 by knowing the halves of 40 and 16. The teacher then used Example B in a similar way. She posed the three related numbers, and students found the halves. She then asked students to look for patterns and relationships between the numbers and the halves to prove what half of 30 is.

Halving numbers like 76 or even 90 can be challenging for students because there is an element of regrouping within the half. This routine can be leveraged for developing skill with those halves by making use of relationships and patterns. In the following figure, four multiples of 10 (60, 70, 80, and 90) are posed. Students are tasked with halving each, but the conversation focuses first on 60 and 80 before students work to find half of 70 and 90. During these discussions, focus their attention on half of 10 being added to or taken from half of 60 or 80 and why. Of course, another strategy is to think of the half (35) between those halves (30 and 40).

C

60	70	80	90

60	70	80	90
30		40	

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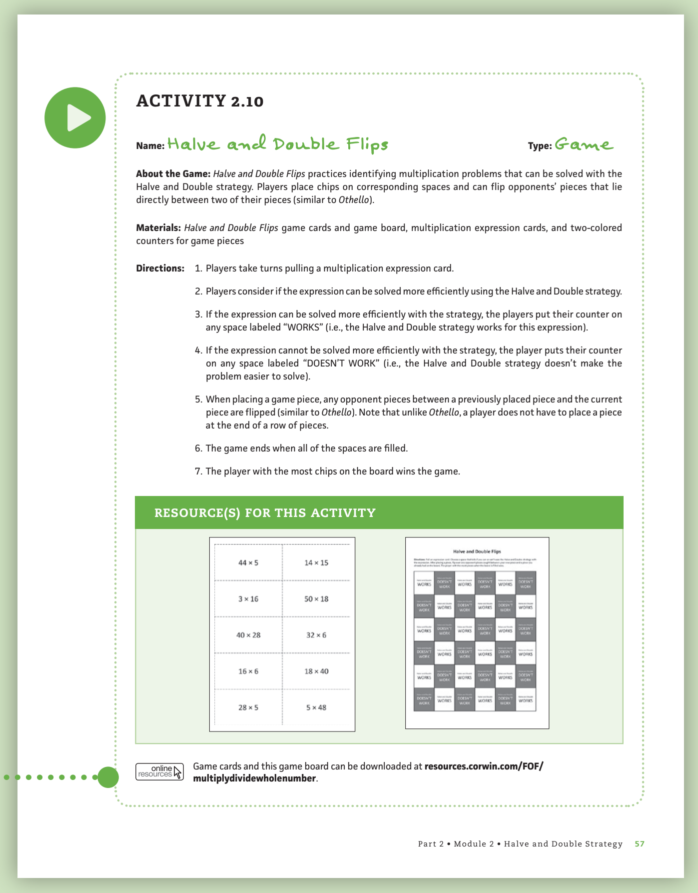
Routines, Games, and Centers for each strategy offer extensive opportunity for student practice.

Download the resources you need for each activity at this book's companion website.



ACTIVITY 2.10

Name: Halve and Double Flips **Type:** Game

About the Game: *Halve and Double Flips* practices identifying multiplication problems that can be solved with the Halve and Double strategy. Players place chips on corresponding spaces and can flip opponents' pieces that lie directly between two of their pieces (similar to *Othello*).

Materials: *Halve and Double Flips* game cards and game board, multiplication expression cards, and two-colored counters for game pieces

Directions:

1. Players take turns pulling a multiplication expression card.
2. Players consider if the expression can be solved more efficiently using the Halve and Double strategy.
3. If the expression can be solved more efficiently with the strategy, the players put their counter on any space labeled "WORKS" (i.e., the Halve and Double strategy works for this expression).
4. If the expression cannot be solved more efficiently with the strategy, the player puts their counter on any space labeled "DOESN'T WORK" (i.e., the Halve and Double strategy doesn't make the problem easier to solve).
5. When placing a game piece, any opponent pieces between a previously placed piece and the current piece are flipped (similar to *Othello*). Note that unlike *Othello*, a player does not have to place a piece at the end of a row of pieces.
6. The game ends when all of the spaces are filled.
7. The player with the most chips on the board wins the game.

RESOURCE(S) FOR THIS ACTIVITY

44 × 5	14 × 15
3 × 16	50 × 18
40 × 28	32 × 6
16 × 6	18 × 40
28 × 5	5 × 48

Halve and Double Flips

WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK
DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS
WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK
DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS
WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK
DOESN'T WORK	WORKS	DOESN'T WORK	WORKS	DOESN'T WORK	WORKS

online resources

Game cards and this game board can be downloaded at **resources.corwin.com/FOF/multiplydividewholenumber**.

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FIGURING OUT

Fluency

MULTIPLICATION & DIVISION

With Whole Numbers

Grades K–8

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FIGURING OUT Fluency

MULTIPLICATION & DIVISION

With Whole Numbers

Grades K–8

A Classroom Companion

John J. SanGiovanni
Jennifer M. Bay-Williams
Rosalba Serrano

CORWIN Mathematics

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Visit the companion website at **resources.corwin.com/FOF/multiplydividewholenumber** for downloadable resources.

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Preface

Fluency is an equity issue. In written documents and in our daily work, we (mathematics teachers and leaders) communicate that every student must be *fluent* with whole number multiplication and division, for example. But we haven't even come close to accomplishing this for each and every student. The most recent National Assessment of Educational Progress (NAEP) data, for example, finds that about two-fifths (41%) of the nation's Grade 4 students are at or above proficient and about one-third (34%) of our nation's Grade 8 students at or above proficient (NCES, 2019). We can and must do better! One major reason we haven't been able to develop fluent students is that there are misunderstandings about what fluency really means.

FIGURING OUT FLUENCY

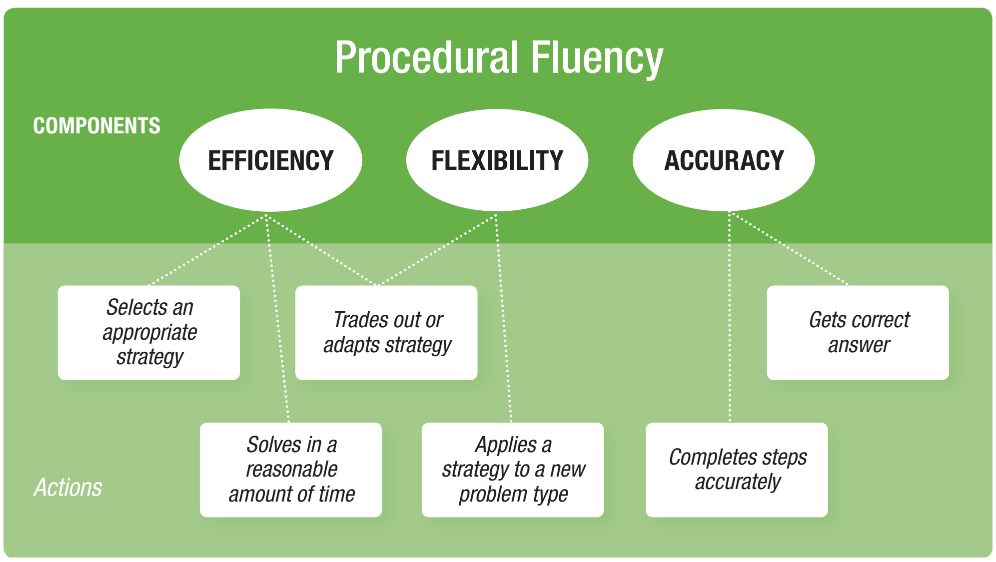
In order to ensure every student develops fluency, we first must:

- Understand what procedural fluency is (and what it isn't),
- Respect fluency, and
- Plan to explicitly teach and assess reasoning strategies.

If you have read our anchor book *Figuring Out Fluency for Mathematics Teaching and Learning*—which we recommend in order to get the most out of *this* classroom companion—you'll remember an in-depth discussion of these topics.

WHAT PROCEDURAL FLUENCY IS AND ISN'T

Like fluency with language, wherein you decide how you want to communicate an idea, fluency in mathematics involves decision-making as you decide how to solve a problem. In our anchor book, we propose the following visual as a way to illustrate the full meaning of fluency.



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Part 1 of this book explains the elements of this procedural fluency graphic. To cut to the chase, procedural fluency is much more than knowing facts and standard algorithms. Fluency involves higher-level thinking, wherein a person analyzes a problem, considers options for how to solve it, selects an efficient strategy, and accurately enacts that strategy (trading it out for another if it doesn't go well). Decision-making is key and that means you need to have good options to choose from. This book provides instructional and practice activities so that students learn different options (Part 2) and then provides practice activities to help students learn to choose options (Part 3).

RESPECT FLUENCY

We are strong advocates for conceptual understanding. We all must be. But there is not a choice here. Fluency relies on conceptual understanding, and conceptual understanding alone cannot help students fluently navigate computational situations. They go together and must be connected. Instructional activities throughout Part 2 provide opportunities for students to discuss, critique, and justify their thinking, connecting their conceptual understanding to their procedural knowledge and vice versa.

EXPLICITLY TEACH AND ASSESS REASONING STRATEGIES

If every student is to be fluent in whole number multiplication and division, then every student needs access to the significant strategies for these operations. And there must be opportunities for students to learn how to select the best strategy for a particular problem. For example, students may learn that the Compensation strategy works well when one of the factors is close to, but less than, a ten or hundred (e.g., 49×8 or 398×7). To accomplish this, all three fluency components must have equitable attention in instruction and assessment. This is a major shift from traditional teaching and assessing, which privileges accuracy over the other two components and the standard algorithm over reasoning strategies.

Let's unpack the phrase *explicit strategy instruction*. According to the *Merriam-Webster Dictionary*, explicit means " fully revealed or expressed without vagueness" ("Explicit," 2021). In mathematics teaching, being explicit means making mathematical relationships visible. A strategy is a flexible method to solve a problem. Explicit strategy instruction, then, is engaging students in ways to clearly see how and why a strategy works. For example, with multiplication, students might compare 12×15 to 6×30 with equal groups (12 groups of 15 as compared to 6 groups of 30) and use rectangles (cutting the rectangle in half and moving the half to double one side) to see how the Halve and Double strategy works. Showing that $12 \times 15 = 6 \times 30$ and proving the generalization of this Halve and Double strategy engages students in higher-level thinking while helping them understand the strategy. For division, students might be asked to explore the conjecture: You can break apart a dividend into a sum, divide it into parts, put it back together and get the same answer. (e.g., For $84 \div 7$, break apart 84 into $70 + 14$. Divide these parts by 7, equaling $10 + 2$, and put them back together to equal 12.) Once understood, students need to explore *when* a

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strategy is a good option. Learning how to use and how to choose strategies *empowers* students to be able to decide how they want to solve a problem, developing a positive mathematics identity and a sense of agency.

USING THIS BOOK

This book is a classroom companion to *Figuring Out Fluency for Mathematics Teaching and Learning.* In that anchor book, we lay out what fluency is, identify the fallacies that stand in the way of a true focus on fluency, and elaborate on necessary foundations for fluency. We also propose the following:

- Seven Significant Strategies across the operations, five of which apply to whole number multiplication and division
- Eight "automaticies" *beyond* automaticity with basic facts, five of which are relevant to whole number multiplication and division
- Five ways to engage students in meaningful practice
- Four assessment options that can replace (or at least complement) tests and that focus on real fluency
- Many ways to engage families in supporting their child's fluency

In Part 1 of this book, we revisit some of these ideas in order to connect specifically to whole number multiplication and division. This section is not a substitute for the anchor book, but rather a brief revisiting of central ideas that serve as reminders of what was fully illustrated, explained, and justified in *Figuring Out Fluency in Mathematics Teaching and Learning*. Hopefully, you have had the chance to read and engage with that content with colleagues *first*, and then Part 1 will help you think about those ideas as they apply to whole number multiplication and division. Finally, Part 1 includes suggestions for how to use the strategy modules.

Part 2 is focused on explicit instruction of each significant strategy for whole number multiplication and division. Each module includes the following:

- An overview for your reference and to share with students and colleagues
- A strategy brief for families
- A series of instructional activities, with the final one offering a series of questions to promote discourse about the strategy
- A series of practice activities, including worked examples, routines, games, and center activities that engage students in meaningful and ongoing practice to develop proficiency with that strategy

As you are teaching and find your students are ready to learn a particular strategy, pull this book off the shelf, go to the related module, and access the activities and ready-to-use resources. While the modules are sequenced in a developmental order overall, the order and focus on each strategy may vary depending on your grade and your students' experiences. Additionally, teaching within or across modules does not happen all at once; rather, activities can be woven into your instruction regularly, over time.

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Part 3 is about becoming truly fluent—developing flexibility and efficiency with multiplication and division of whole numbers. Filled with more routines, games, and centers, the focus here is on students *choosing* to use the strategies that make sense to them in a given situation. Part 3 also provides assessment tools to monitor students' fluency. As you are teaching and find your students are needing opportunities to choose from among the strategies they are learning, pull this book off the shelf and select an activity from Part 3.

In the Appendix, you will find lists of all the activities in order to help you easily locate what you are looking for by strategy or by type of resource.

This book can be used to complement or supplement any published mathematics program or district-created program. As we noted earlier, elementary mathematics has tended to fall short in its attention to efficiency and flexibility (and the related Fluency Actions illustrated in the earlier graphic). This book provides a large collection of activities to address these neglected components of fluency. Note that this book is part of a series that explores other operations and other numbers. You may also be interested in *Figuring Out Fluency for Whole Number Addition and Subtraction* as well as the classroom companion books for decimal and fraction operations.

WHO IS THIS BOOK FOR?

With 120 activities and a companion website with resources ready to download, this book is designed to support classroom teachers as they advance their students' fluency with whole number multiplication and division. Special education teachers will find the explicit strategy instruction, as well as the additional practice, useful in supporting their students. Mathematics coaches and specialists can use this book for professional learning and to provide instructional resources to the classroom teachers they support. Mathematics supervisors and curriculum leads can use this book to help them assess fluency aspects of their mathematics curriculum and fill potential gaps in resources and understanding. Teacher preparation programs can use this book to galvanize preservice teachers' understanding of fluency and provide teacher candidates with a wealth of classroom-ready resources to use during internships and as they begin their career.

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Acknowledgments

Just as there are many components to fluency, there are certainly many components to having a book like this come to fruition. The first component is the researchers and advocates who have defined procedural fluency and effective practices that support it. Research on student learning is hard work, as is defining effective teaching practices, and so we want to begin by acknowledging this work. We have learned from these scholars, and we ground our ideas in their findings. It is on their shoulders that we stand. Second are the teachers and their students who have taken up "real" fluency practices and shared their experiences with us. We would not have taken on this book had we not seen firsthand how a focus on procedural fluency in classrooms truly transforms students' learning and shapes their mathematics identities. It is truly inspiring! Additionally, the testimonies from many teachers about their own learning experiences as students and as teachers helped crystalize for us the facts and fallacies in this book. A third component to bringing this book to fruition was the family support to allow us to actually do the work. We are all grateful to our family members—expressed in our personal statements that follow—who supported us 24/7 as we wrote during a pandemic.

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gone above and beyond as a partner in the work, ensuring that our ideas are as well stated and useful as possible. The entire editing team at Corwin has been creative, thorough, helpful, and supportive.

As with fluency, no component is more important than another, and without any component, there is no book, so to the researchers, teachers, family, and editing team, thank you. We are so grateful.

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PART 1

FIGURING OUT FLUENCY

Key Ideas

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WHAT IS FLUENCY WITH WHOLE NUMBER MULTIPLICATION AND DIVISION?

To set the stage for figuring out fluency for whole number multiplication and division, take a moment to do some math. Find the solution to each of these.

35×19	$2{,}458 \div 8$	$756 \div 7$
320×5	$240 \div 3$	24×15

How did you find the products and quotients? Did you use the same approach or strategy for each? Did you move between different strategies? Did you change out a strategy based on the numbers within the problem? Did you start with one strategy and shift to another? You likely said "yes" to all of these questions because you fluently multiply and divide whole numbers. Yet another reader can answer "yes" to each of these questions as well but solve each problem differently. This is true because fluency is a way of thinking rather than a way of doing. Thinking is unique to each individual. Thinking is grounded in parameters but its execution is left to the understanding, preference, and creativity of each thinker.

Of course, there are strategies used most frequently for certain problems (based on the numbers in the problem), but even in those examples, efficient alternatives are likely. For example, 320×5 may seem like a problem that "fits" Partial Products (Module 4). Find the product of 300×5 (1,500) and 20×5 (100) and add them together (1,600). But a Halve and Double approach (Module 2) may "jump out" to another person who sees 320×5 as 160×10. Which method is closer to your way of thinking? Or did you think about it differently?

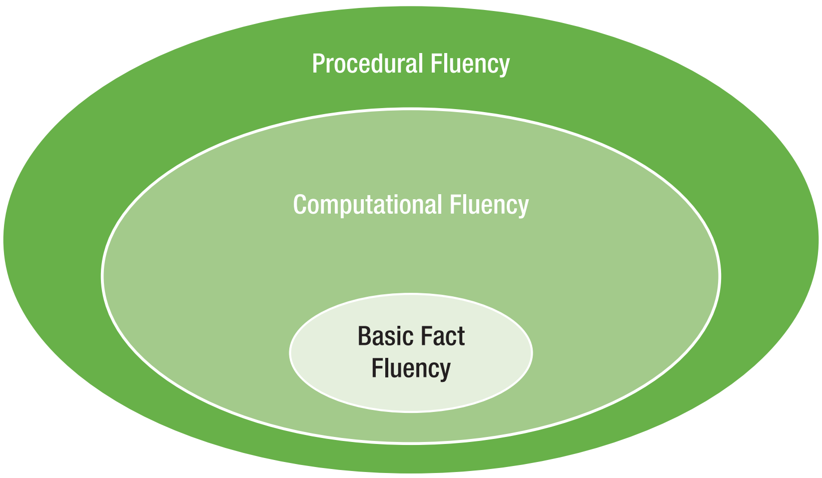
Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy. Real fluency is not the act of replicating someone else's steps or procedures for doing mathematics. It is the act of thinking, reasoning, and doing mathematics on one's own. Before fluency can be taught well, you must understand what fluency is and why it matters.

TEACHING TAKEAWAY

Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy.

Procedural fluency is an umbrella term that includes basic fact fluency and computational fluency (see Figure 1). *Basic fact fluency* attends to fluently adding,

FIGURE 1 ● The Relationship of Different Fluency Terms in Mathematics



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subtracting, multiplying, and dividing single-digit numbers. *Computational fluency* refers to the fluency in four operations across number types (whole numbers, fractions, etc.), regardless of the magnitude of the number. Procedural fluency encompasses both basic fact fluency and computational fluency, plus other procedures like finding equivalent fractions.

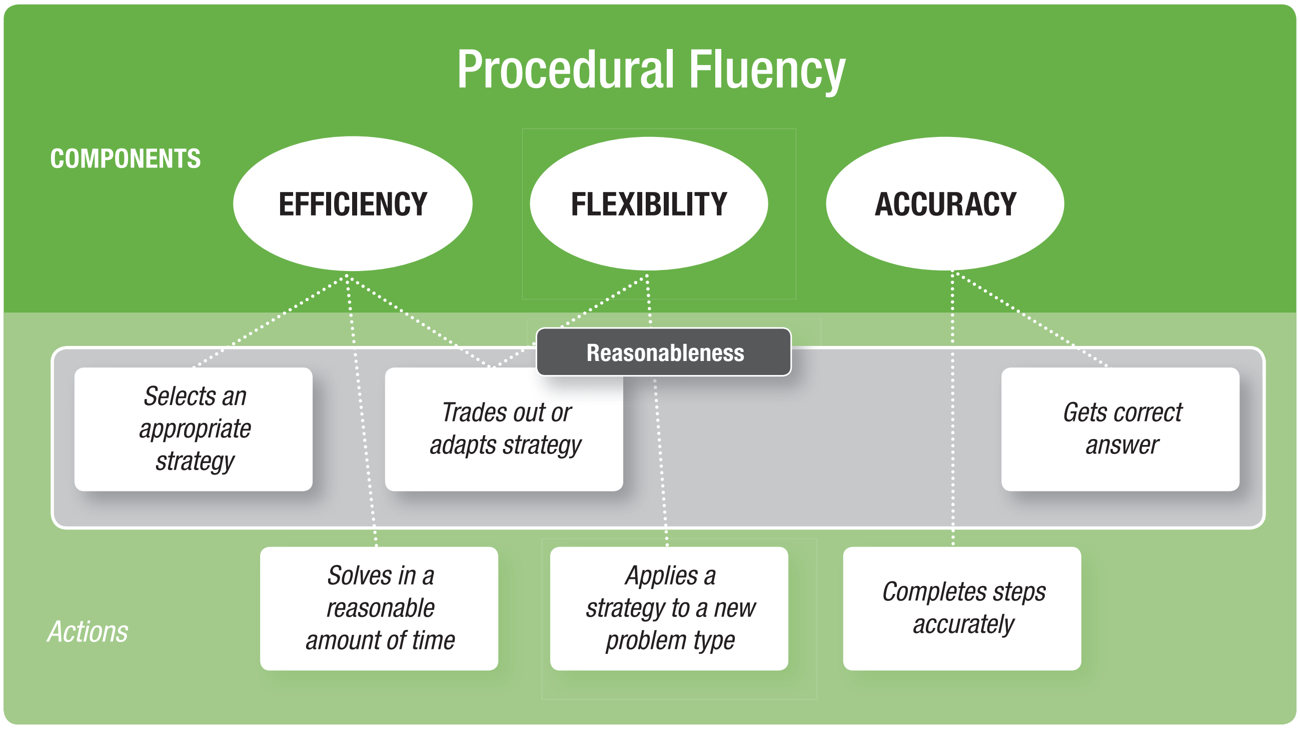
Beyond being an umbrella term that encompasses basic fact and computational fluency, procedural fluency is well defined as solving procedures efficiently, flexibly, and accurately (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics, 2014). These three **components** are defined as follows:

> *Efficiency:* solving a procedure in a reasonable amount of time by selecting an appropriate strategy and readily implementing that strategy
>
> *Flexibility:* knowing multiple procedures and can apply or adapt strategies to solve procedural problems (Baroody & Dowker, 2003; Star, 2005)
>
> *Accuracy:* correctly solving a procedure

Strategies are not the same as algorithms. Strategies are general methods that are flexible in design, compared to algorithms that are established steps implemented the same way across problems.

To focus on fluency, we need specific observable actions that we can look for in what students are doing in order to ensure they are developing fluency. We have identified six such actions. The three components and six Fluency Actions, and their relationships, are illustrated in Figure 2.

FIGURE 2 ● Procedural Fluency Components, Actions, and Checks for Reasonableness



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Three of the six Fluency Actions (should) attend to reasonableness. Fluency Actions and reasonableness are described later in Part 1, but first, it is important to consider why this "bigger" (comprehensive) view of fluency matters.

WHY FOCUS ON FLUENCY FOR WHOLE NUMBER MULTIPLICATION AND DIVISION?

There are two key reasons why it's important to focus on fluency with whole number multiplication and division. First, it is a critical foundation for ensuring that students fully realize procedural fluency in general, with all kinds of numbers. Fluency with whole numbers begins with developing fluency with single-digit multiplication and division (the basic facts) and seeing how strategies such as Subtract-a-Group can be transferred to using Compensation in order to multiply 29×8 efficiently (e.g., 29×8 is adjusted to 30×8 and to compensate, 8 is subtracted from the product). Basic facts are addressed in more detail later in Part 1.

Second and most importantly, developing fluency is an equity issue. Equipping students with options for how to solve multiplication and division problems and positioning students to choose a method that works best for them develops a positive mathematics identity and sense of agency. Conversely, trying to remember algorithms and feeling anxiety about being correct or fast develops a negative mathematics identity and lack of agency.

WHAT DO FLUENCY ACTIONS LOOK LIKE FOR WHOLE NUMBER MULTIPLICATION AND DIVISION?

The six Fluency Actions are observable and therefore form a foundation for assessing student progress toward fluency. Let's take a look at what each of these actions looks like in the context of whole number multiplication and division.

FLUENCY ACTION 1: Select an Appropriate Strategy

TEACHING TAKEAWAY

Selecting *an* appropriate strategy does not mean selecting *the* appropriate strategy. Many problems can be solved efficiently in more than one way.

Selecting *an* appropriate strategy does not mean selecting *the* appropriate strategy. Many problems can be solved efficiently in more than one way. Here is our operational definition: Of the available strategies, the one the student opts to use gets to a solution in about as many steps and/or about as much time as other appropriate options.

Consider 25×38. One could leverage the distributive property by breaking apart factors and adding the partials as shown in Figure 3a, in which 25 is decomposed into 10 + 10 + 5 and the products of those numbers and 38 are added together. One could break apart 38 by factors using 2×19 and rethink 25×38 as $25 \times 2 \times 19$, which becomes 50×19 (as shown in Figure 3b). Figure 3c shows Compensation in which one thinks of a friendlier problem, $25 \times 40 = 1{,}000$, and then compensates for the two extra groups of 25 by taking them from 1,000. Notice that this Fluency Action is one of three connected to *reasonableness*. Within this action is noticing when a strategy "fits" the numbers in the problem. This particular problem fits well with Compensation because 38

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is close to (but less than) 40. The other strategies are reasonable if you are adept at multiplying by 5s. In other words, what is an "appropriate" strategy depends on the problem and on the person (their knowledge and experiences).

FIGURE 3 ● Multiplying 25 × 38

a. Break Apart to Multiply

(break apart into addends)

25×38

$10 \times 38 = 380$
$10 \times 38 = 380$
$5 \times 38 = 190$
950

b. Break Apart to Multiply

(break apart into factors)

25×38

$25 \times 2 \times 19$
$50 \times 19 = 950$

c. Compensation

(change a factor and adjust the answer)

25×38

$25 \times 40 = 1{,}000$
$25 \times 2 = 50$
$1{,}000 - 50 = 950$

A strategy cannot be used until it is understood. Once understood, a strategy becomes part of a student's repertoire of options and they are then able to select the strategy.

Importantly, we name strategies so that we can talk about them. But one approach may fit within various types of strategies and have different names. For example, solving 23 × 9 as 20 × 9 + 3 × 9 may be using Break Apart thinking or Partial Products thinking. The focus must be on the ideas (not the naming of the strategy).

FLUENCY ACTION 2: Solve in a Reasonable Amount of Time

There is no set amount of time that should be expected for solving any whole number multiplication or division problem. Students should be able to work through a problem without getting stuck or lost. The amount of time is relative to the student's grade and mathematical maturity. Keep in mind that appropriate strategies can be carried out in inefficient, unreasonable ways. For example, Figure 3a shows a Break Apart approach that can be enacted in a reasonable amount of time. In contrast, Figure 4 shows this approach with many more parts, which requires significantly more time. The approach is likely the result of the student's ability to recall or find certain products (×10, ×2). While this may be an appropriate beginning strategy, it is ultimately not an approach that can be completed in a reasonable amount of time.

FIGURE 4 ● Solving 25 × 38 by Breaking Apart 38

25×38

$10 \times 25 = 250$
$10 \times 25 = 250$
$10 \times 25 = 250$
750

$2 \times 25 = 50$
$2 \times 25 = 50$
$2 \times 25 = 50$
$2 \times 25 = 50$
200

$25 \times 38 = 950$

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FLUENCY ACTION 3: Trade Out or Adapt a Strategy

As students' number sense and understanding of strategies advances, they are able to adapt and trade out strategies. Let's revisit 25×38. A student might attempt 25×38 as illustrated in Figure 3a but get stuck because they don't know 5×38. Thus, they decide to *adapt* the strategy by breaking apart 38 instead:

$$25 \times 30 = 750$$
$$25 \times 8 = 200$$
$$750 + 200 = 950$$

Or a student might *trade out* the Break Apart strategy for another strategy. For example, the student might use Compensation (see Figure 3c) or Partial Products (two options are illustrated in Figure 5a). Notice that this Fluency Action is one of three connected to *reasonableness*. Within this action is noticing how the use of the strategy is going. If it isn't going well or if a student is getting bogged down, then the strategy needs to be adapted or traded for another, more efficient option.

FIGURE 5 ● Partial Products

a. Partial Products Decomposing Both Factors

25×38

$20 \times 30 = 600$
$5 \times 30 = 150$
$20 \times 8 = 160$
$5 \times 8 = 40$
950

b. Partial Products Decomposing One Factor

25×38

$30 \times 25 = 750$
$8 \times 25 = 200$
950

FLUENCY ACTION 4: Apply a Strategy to a New Problem Type

The Compensation work shows how, when multiplying 25×38, 38 can be thought of as 40 to find $25 \times 40 = 1{,}000$ and then two groups of 25 can be taken away. That individual's first experience with compensation was likely finding products of $\times 9$ basic facts that transferred to multidigit factors that were one group away from a friendly computation such as 18×9 or 35×39. In time, this strategy will be applied to multiplying fractions by thinking of $6 \times 2\frac{3}{4}$ as 6×3, taking away $6 \times \frac{1}{4}$ $(18 - 1\frac{1}{2} = 16\frac{1}{2})$ or multiplying decimals by thinking of 6×1.9 as 6×2 and taking away six tenths $(12 - 0.6 = 11.4)$.

FLUENCY ACTIONS 5 AND 6: Complete Steps Accurately and Get Correct Answers

These two Fluency Actions are about accuracy. An error at the end of a problem may be due to an error in how a strategy was enacted. For example, in

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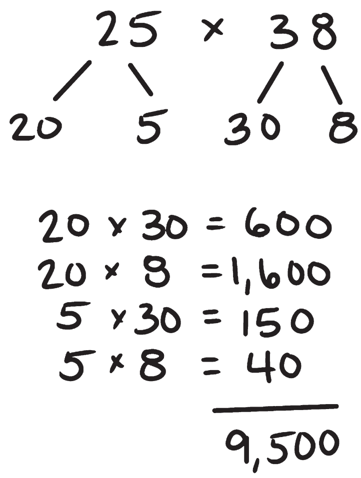
using Compensation for 25 × 38, a student changes the problem to 25 × 40 to get 1,000 and then *subtracts* 2 from the answer, resulting in the answer 998. This incorrect answer is due to a misconception of the steps in implementing the Compensation strategy (specifically not recognizing that they need to compensate by subtracting two groups of 12). Conversely, a student may enact the strategy accurately but make a computational error. The student's work in Figure 6 is a good example of this. Here, the student correctly enacts the steps for Partial Products but makes an error with 20 × 8 that leads to the wrong answer.

FIGURE 6 ● Incorrect Partial Product Example



Fluency Action 6 is one of three connected to *reasonableness*. Within this action is noticing if your answer makes sense. While reasonableness has been woven into the discussion of Fluency Actions, it is critical to fluency and warrants more discussion.

REASONABLENESS

As described earlier, reasonableness is more than "checking your answer." Reasonableness occurs in three of the six Fluency Actions as shown in Figure 2 and described within the related Fluency Actions. Let's revisit 25 × 38. This example has several reasonable options, including Break Apart to Multiply, Compensation, and Halve and Double (Action 1). Skip-counting by 25s or 38s is not a reasonable strategy. Let's say a student chooses Break Apart (using addends). They break apart 25 (20 × 38 + 5 × 38) but don't know these partial products. The student recognizes that this attempt is not going well and adapts or trades out the strategy (Action 3). Finally, they notice that 950 is a reasonable answer (Action 6). Knowing that 25 × 38 = 950 is reasonable can be determined in (at least) three different ways:

1. You might estimate the product to be less than 1,200 because both factors round up to 30 and 40, respectively.
2. You might estimate it to be about 1,000 using friendly numbers (25 × 40).
3. You might think about it being in the range of 600 (20 × 30) and 1,200 (30 × 40).

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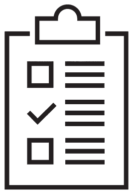
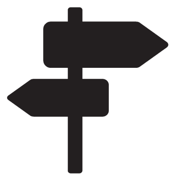
It takes time to develop reasonableness. It should be practiced and discussed as often as possible. Students can develop reasonableness by practicing three moves (a match to the Fluency Actions 1, 3, and 6).

THREE 'Cs' OF REASONABLENESS

Choose: Choose a strategy that is efficient based on the numbers in the problem.
Change: Change the strategy if it is proving to be overly complex or unsuccessful.
Check: Check to make sure the result makes sense.

You can encourage and support student thinking about reasonableness by providing Choose, Change, Check reflection cards (see Figure 7). These cards can be adapted into anchor charts for students to use while working on problems or during class discussions about multiplying and dividing.

FIGURE 7 ● Choose, Change, Check Reflection Card for Students

CHECKS FOR REASONABLENESS		
Choose	Change	Check
		
Is this something I can do in my head? What strategy makes sense for these numbers?	Is my strategy going well or should I try a different approach? Does my answer so far seem reasonable?	Is my answer close to what I anticipated it might be? How might I check my answer?

Icon sources: Choose by iStock.com/Enis Aksoy; Change by iStock.com/Sigit Mulyo Utomo; Check by iStock.com/Indigo Diamond

online resources This resource can be downloaded at **resources.corwin.com/FOF/multiplydividewholenumber**.

WHAT FOUNDATIONS DO STUDENTS NEED TO DEVELOP FLUENCY WITH MULTIPLICATION AND DIVISION?

To develop fluency, students need a strong foundation in five domains:

- *Conceptual understanding*: knowing the meaning of the operations
- *Properties*: being able to use the operations in order to manipulate numbers and retain equivalence
- *Utilities*: small skills that make a big difference, such as knowing or easily finding half of a number
- *Computational estimation*: being able to quickly and flexibly determine a "close" answer
- *Basic facts*: single-digit addition, subtraction, multiplication, and division facts that are needed for multidigit work

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Rushing students to strategy instruction before these foundations are firmly in place can be disastrous.

CONCEPTUAL UNDERSTANDING

Developing fluency from conceptual understanding begins with including concrete experiences for students so they can make sense of the quantities. Hence, developing fluency *begins* with stories. It is a mistake to save story problems as an application, as stories give students a context from which they can reason.

TEACHING TAKEAWAY

Developing fluency *begins* with stories and contexts. It is a mistake to save story problems as an application, as stories give students a context from which they can reason.

As students work with multiplication and division story problems, there are few important ideas to keep in mind:

1. Stories need to be *relevant* to students, meaning that students are familiar with the context and it is interesting to students.
2. Avoid key-word strategies for solving story problems so that students must make sense of the problem and how multiplication or division can be used to find a solution.
3. Balance experiences with division between both partitive and quotative (measurement) interpretations of division.
4. Vary and balance experience with the different situations for multiplication and division as shown in Figure 8.
5. Use representations that are true to the situation (e.g., equal group representations for equal group problems, area representations for area problems) until students have a deep understanding and are able to work symbolically and move between representations and tell explicitly how they are related.

Figure 8 can be used as a resource to be sure you are varying your story types (for example, you can tally which types of stories you are telling as part of action research).

FIGURE 8 ● Multiplication and Division Situations

MULTIPLICATION AND DIVISION SITUATIONS			
Equal Groups	**Area/Array**	**Compare**	**Combinations**
Story is about a quantity of same-sized grouped amount.	Story is about a quantity of equal-length rows.	Story compares two quantities multiplicatively.	Story is about finding how many pairings are possible (and beyond).
Ex: AJ has ____ stacks of books. Each stack has ____ books. She has a total of ____ books.	AJ has ____ books on each shelf. She has ____ shelves. She has a total of ____ books.	AJ has ____ books. Ian has ____ books. AJ has ____ times more books than Ian.	AJ has ____ books and ____ magazines. She takes one of each to school. There are ____ different combinations.

In addition to stories, visuals provide concrete, conceptual beginnings for students. For example, young children count collections of objects in order to start learning to skip count (Franke, Kazemi, & Turrou, 2018). Counting objects progresses to counting visuals and representations, which eventually leads to abstract counting strategies and representation of groups.

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PROPERTIES AND UTILITIES FOR STRATEGIC COMPETENCE

In addition to conceptual foundations, fluency is grounded in using properties of the operations and a few other skills that we refer to as "utilities" because students must utilize them in their reasoning. First, fluency with multiplication relies heavily on students using the commutative, associative, and distributive properties of multiplication. Note that knowing properties does not equal using properties. It is *not* useful to have students simply name the associative property. It is absolutely necessary that students *utilize* this property in solving problems efficiently. For example, in the following problem, students break apart factors (44) by their factors (4 × 11) and rearrange these numbers (mentally or in writing) using the commutative property to create friendly computations.

TEACHING TAKEAWAY

Knowing properties does not equal using properties.

$$44 \times 25$$

$$4 \times 11 \times 25$$
$$4 \times 25 \times 11$$
$$100 \times 11$$
$$1{,}100$$

The distributive property is most critical for fluency with multiplication because so many strategies, including the standard algorithm, make use of it. On the left in the following example, the distributive property is applied to 8 × 398 in order to find partial products. Numerically, this is 8 × 398 = 8 (300 + 90 + 8). On the right, the distributive property is used for a compensation strategy. Numerically, this is 8 × 398 = 8(400 + –2) = 8(400 – 2).

$$8 \times 398$$

	300	90	8
8	2,400	720	64

3,184

$$8 \times 398$$

$$8 \times 400 = 3{,}200$$
$$-8 \times 2 = 16$$
$$8 \times 398 = 3{,}184$$

See Chapter 3 (pp. 47–75) of *Figuring Out Fluency* for more about foundations and good beginnings for fluency.

Beyond the properties is a short list of utilities that support fluency, which is presented in Figure 9.

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FIGURE 9 ● Utilities for Strategic Competence With Multiplication and Division

UTILITY	WHAT IT IS	RELATIONSHIP TO FLUENCY
Distance From a 10	Knowing that 9 is 1 away from 10 (and 8 is 2 away and so on).	Knowing how far a number (e.g., 5, 6, 7, 8, 9) is from 10 is necessary for finding how many groups of a number are away from a friendly computation (e.g., 28 is 2 away from 30, so 28×6 is two groups of 6 away from 30 groups of 6).
Composing and Decomposing Numbers Flexibly	Understanding diverse, flexible ways to compose and decompose, including but not limited to place value decomposition.	Flexibly decomposing numbers supports strategy selection and facility with any of the strategies.
Skip Counting	Skip counting by multiples of tens, hundreds, and thousands as well as other useful benchmarks such as 25.	Efficiency comes from skip-counting with multiples. This applies to all multiplication and division strategies.
Multiplying and Dividing by Tens, Hundreds, and Thousands	Understanding that 6×40 is similar to 6×4, in that 6×40 is saying six groups of 4 tens, which is 24 tens or 240.	Flexible decomposition of factors relies on recognition and knowing of products of factors that are multiples of 10, 100, or 1,000.

COMPUTATIONAL ESTIMATION

Just like computation, there are strategies for estimation and the use of those strategies should be *flexible*. For multiplication and division of whole numbers, students might use any of these methods:

1. *Rounding*: Flexible rounding means that one or both numbers might be rounded. Students may round to the nearest number or they may round one number up and one number down to have a more accurate estimate. Rounding is a well-known strategy but is often approached in a step-by-step manner, which can interfere with the point of estimating—getting a quick idea of what the answer will be close to. Use conceptual language, such as, “Which tens/hundreds/thousands is that number close to?” Help students understand that they choose how to round. For example, for 24×34, rounding both to the nearest 10 will give a low estimate, whereas rounding one up and one down gives a closer estimate.
2. *Front-end estimation*: In its most basic form, students just multiply or divide using the largest place value. More flexibly, though, students may use the largest two place values or adjust their estimate because of what they notice with the rest of the numbers. Front-end estimation is *quick*. For example, 74×55 is about 3,500 (relying on 70×50). To adjust, take a quick look to the right and decide to keep estimate or adjust. With division, that front end must focus on compatibles. For example, to estimate $4{,}576 \div 7$, the front end is altered to be 4,200 or 4,900 because these “fronts” are multiples of 7 (this is an overlap to the next strategy, compatible numbers!).
3. *Compatible numbers*: With flexibility in mind, students change one or both of the numbers to a nearby number so that the numbers are easy to multiply or divide. For example, estimating the quotient of $337 \div 72$ could be to think of 337 as 350 and 72 as 70 to estimate a quotient of about 5. Compatibles are particularly useful with division. Consider estimating $345 \div 8$. Compatible alternatives might be $320 \div 8$, $400 \div 8$, or $350 \div 7$.

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DEVELOPING BASIC FACT FLUENCY

The teaching of basic facts must attend to conceptual understanding and strategies for reasoning rather than rote instruction (Bay-Williams & Kling, 2019; O'Connell & SanGiovanni, 2014). Reasons to focus on *strategies* when teaching the basic facts (as opposed to memorizing) include the following:

1. It is well established across many studies that students actually learn and retain their facts better when they focus on conceptual understanding versus memorization. In fact, students don't just learn and retain their facts better, they perform better in math *in general* (e.g., Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Brendefur, Strother, Thiede, & Appleton, 2015; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008; Purpura, Baroody, Eiland, & Reid, 2016).
2. Students need to know and use these strategies to support whole number multiplication and division (as well as decimal and fraction operations). In our *Figuring Out Fluency* anchor book, we elaborate more on the key strategies and ideas for effectively developing basic fact fluency.
3. Students who learn to use and choose strategies for basic facts develop confidence. Students who memorize often develop anxiety. A student who knows how to generate an answer to a quotient such as 9 ÷ 6 (beyond counting) doesn't have to worry if they forget the fact. This sense of agency is critical to student success in mathematics!

Figure 10 lists the basic fact strategies for multiplication and division. To be clear, automaticity is the goal for learning basic facts. Students become automatic through learning the strategies and practicing them over and over again. In so doing, students develop automaticity with the facts *and with implementing the strategies*. Examples are illustrated in Figure 11. Keep in mind that there are many ways to implement a reasoning strategy, and only one way is shown for each example.

FIGURE 10 ● Reasoning Strategies for Basic Fact Multiplication and Division

STRATEGY NAME	HOW THE STRATEGY WORKS	EXAMPLE STUDENT TALK
Multiplication	**Example: 6 × 7**	
Doubling	Student sees an even factor, finds the product of half of that factor, and doubles their answer.	I got 42. I know 3 times 7 is 21 and I doubled 21.
Add-a-Group	Student thinks of a known fact where one of the factors is one less, multiplies, then adds a group back on.	When I see a 6, I use my 5s: 5 times 7 is 35 and 7 more is 42.
Subtract-a-Group	Student thinks of a known fact where one of the factors is one more, multiplies, then adds a group back on.	I know 7 groups of 7 is 49, so I subtract one group of 7 and I have 42.
Near Squares	Student uses a square fact they know and then adds or subtracts a group. *Note: This is an undertaught but useful strategy.*	Well, 6 times 6 is 36, and I add 6 more and get 42.
Division	**Example: 36 ÷ 9**	
Think Multiplication	Student thinks, *How many groups of 9 make 36?*	I know 9 times 4 is 36, so it's 4. Or I used doubling to get to 18, doubled again, and got 36, so it is 4.

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Basic fact strategies evolve directly to the significant reasoning strategies for multidigit multiplication and division, which are the focus of Part 2 of this book. You can see this transformation in the examples in Figure 11.

FIGURE 11 ● How Basic Fact Strategies Grow Into General Reasoning Strategies

REASONING STRATEGY	EXAMPLES WITH MULTIDIGIT NUMBERS
Add-a-Group(s) leads to **Break Apart to Multiply**, and **Partial Products** strategies	$42 \times 6 \rightarrow (40 \times 6) + (2 \times 6)$
Subtract-a-Group(s) leads to **Compensation**	$79 \times 5 \rightarrow (80 \times 5) - (1 \times 5)$
Doubling applies to the **Halve and Double** strategy and extends to **Break Apart to Multiply**, involving breaking apart into factors	$36 \times 5 \rightarrow 18 \times 10$ $45 \times 6 \rightarrow 45 \times 2 \times 3 \rightarrow 90 \times 3$ $25 \times 68 \rightarrow 25 \times 4 \times 17 \rightarrow 100 \times 17$
Think Multiplication extends to larger numbers and to **Computational Estimation** involving division.	$49 \div 7 \rightarrow 490 \div 7$ [or $490 \div 70$] $4871 \div 85 \rightarrow$ About how many 8s are in 48? [6, so estimate is 60]

WHAT AUTOMATICITIES DO STUDENTS NEED BEYOND THEIR BASIC FACTS?

Unlike the foundation of conceptual understanding, automaticities are not prerequisites for, but coincide with, strategy instruction. For example, automaticity with basic facts (just discussed) begins with strategy instruction and leads to eventual automaticity with the facts. But there are automaticities beyond the basic facts that support student reasoning!

Automaticity is the ability to complete a task with little or no attention to process. Little thought, if any, is given to skills that are automatic (Cheind & Schneider, 2012). We consider automaticities to be those skills that a fluent person can do without much attention to process. For example, you know that four 25s are 100 and a drawing or repeated addition is not needed; it is intuitive or reflexive. Figure 12 identifies automaticities that are particularly critical for multiplying and dividing whole numbers. Of course, this is not a complete list. These automaticities are strengthened through strategy instruction (and conversely, having these automaticities strengthens students' capacities to use strategies).

See Chapter 5 (pp. 107–129) of *Figuring Out Fluency* for more about automaticities for fluency.

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FIGURE 12 ● Automaticities for Multiplying and Dividing Whole Numbers

AUTOMATICITY	WHAT IT IS	HOW IT COMPLEMENTS STRATEGY INSTRUCTION
Basic facts	Quickly recognizing how a problem relates to a basic fact (e.g., 30 × 80 relates to 3 × 8).	Identifying relationships to basic facts helps students consider which numbers to decompose and how to decompose them.
Using 25s	Knowing multiples of 25 (within reason) and how they are related to multiples of 250, 2,500, and so on.	This helps students think about how to decompose factors and find partials when multiplying and dividing.
Using 15s and 30s	Knowing multiples of 15 and 30 and ways to decompose them efficiently (e.g., 75 is 60 and 15 or two 30s and 15 or five 15s).	This helps students think about how to decompose factors and find partials when multiplying and dividing.
Doubling	Doubling a given number.	Although explicitly connected to the Halve and Double strategy, Doubling is useful for Compensation and strategies with partials.
Halving	Finding a half of a given number.	Though explicitly connected to the Halve and Double strategy, Halving is useful for Compensation and strategies with partials.

WHAT ARE THE SIGNIFICANT STRATEGIES FOR MULTIPLYING AND DIVIDING WHOLE NUMBERS?

Teaching strategies beyond the common algorithms has been a challenge, as there has been pushback and criticism from families and in social media. Two questions require attention:

1. Why do students need strategies when they can use the standard algorithm?

One way to quickly respond to this question is to share an example for which the standard algorithm takes much more time than an alternative. Examples include 24 × 5, 125 ÷ 5, or a problem that has various possibilities, like 16 × 35. Why learn other methods? Because many problems can be solved more efficiently another way. Fluent students look for efficient methods; if students are limited in the methods they are taught, they have little to choose from, which limits flexibility and efficiency.

TEACHING TAKEAWAY

Students don't need to constantly learn dozens of new strategies, but rather connect how the key strategies they learned for basic facts are transferred to other numbers.

2. What strategies are worthy of attention?

Let's just take some pressure off here. The list is short, and we must help students see that they are not necessarily learning a *new* strategy, but they are applying a strategy they learned with basic facts and transferring it to other numbers. In *Figuring Out Fluency* we propose Seven Significant Strategies. Of these, five relate to multiplying and/or dividing whole numbers and they are listed in Figure 13.

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FIGURE 13 ● Reasoning Strategies for Multiplying and Dividing Whole Numbers

REASONING STRATEGIES	RELEVANT OPERATIONS
1. Break Apart to Multiply (Module 1)	Multiplication
2. Halve and Double (Module 2)	Multiplication
3. Compensation (Module 3)	Multiplication
4. Partial Products and Quotients (Modules 4 and 6)	Multiplication and Division
5. Think Multiplication (Module 5)	Division

Using an area model or a number line is *not* a strategy. It is a representation. If a student multiplies using an area model, they are implementing a strategy—perhaps Break Apart or Partial Products. When a student says, "I used an area model," ask *how* they used it—then you will learn what strategy they used. Teaching for fluency means that each of these strategies is explicitly taught to students. We teach students to *use* the strategy, and then we give students many opportunities to engage in *choosing* strategies (Part 3 of this book). Explicitly teaching a strategy does not mean turning the strategy into an algorithm. Strategies require flexible thinking. Each module provides instructional ideas and practice to ensure students become adept at using each strategy flexibly. There is also a module on **standard algorithms** (Module 7), so that they are integrated into the use of strategies.

HOW DO I USE THE PART 2 MODULES TO TEACH, PRACTICE, AND ASSESS STRATEGIES?

Part 2 is a set of modules, each one focused on understanding why a specific reasoning strategy works and learning how to use it well. The seven modules in Part 2 each have a consistent format. First, each module provides an overview of the strategy—unpacking what it is, how it works, and when it is useful. Then, each module provides a series of instructional activities for explicit strategy instruction, followed by a collection of practice activities, including routines, games, and centers.

EXPLICIT STRATEGY INSTRUCTION

Strategies must be explicitly taught so that students understand them and can use them. Each module provides core teaching activities for explicit instruction. The activities are designed so that you can modify and extend them as needed. Any one activity might form the focus of your instruction over the course of multiple lessons. Keep in mind that you can swap out tools and representations as well as adjust the numbers within the task.

The last teaching activity in each module is a collection of *investigation prompts* that you can use to develop reasoning and understanding of the strategy. Each investigation prompt itself can easily become a core teaching task.

We intend for students to work with instructional activities in collaborative partner or group settings. We encourage you to let students make their own

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meaning and to make mistakes. After students engage in the activities, a group discussion is needed to focus student thinking on the concepts within the strategy, how the strategy works, and the different ways a strategy might be carried out.

QUALITY PRACTICE

Students need access to quality practice that is not a worksheet. Quality practice is focused on a strategy, varied in type of engagement, processed by the student to make sense of what they did, and connected to what they are learning.

See Chapter 6 (pp. 130– 153) of *Figuring Out Fluency* for more about quality practice.

Each practice section begins with **worked examples**. Worked examples are opportunities for students to attend to the thinking involved with a strategy, without solving the problem themselves. We feature three types to get at all components of fluency:

1. *Correctly worked example*: efficiency (selects an appropriate strategy) and flexibility (applies strategy to a new problem type)
2. *Partially worked example*: efficiency (selects an appropriate strategy) and accuracy (completes steps accurately; gets correct answer)
3. *Incorrectly worked example*: accuracy (completes steps accurately; gets correct answer)

Also, comparing two correctly worked examples is very effective in helping students learn to choose efficient methods. Throughout the modules are dozens of examples, which can be used as worked examples (and adapted to other similar worked examples). Your worked examples can be from a fictional "student" or authentic student work. Some of the prompts from teaching the strategy section are, in fact, worked examples.

The remaining practice activities include routines, games, and centers. Each activity provides a brief "About the Activity" statement to help you quickly match what your students need with a meaningful activity. Game boards, recording sheets, problem cards, and all other activity resources, including modified versions, are available as online resources that you can download, modify, print, and copy. General resources, including number cards, mini ten-frame cards, multiplication charts, and more, are also available for download on the companion website.

ASSESSING STRATEGY USE

Each module offers a plethora of practice activities. As students are practicing, you can observe and assess the extent to which they are able to apply the selected strategy. **Observation tools** help you keep track of where each student is and monitor their progress. An observation tool can be simple, such as a class list with an extra column. Your observations can be codes:

+ Is regularly implementing the strategy adeptly

✔ Understands the strategy, takes time to think it through

– Is not implementing the strategy accurately

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A note-taking observation tool provides space for you to insert notes about how a student is doing (see Figure 14). You can laminate the tool and use dry-erase markers to reuse it for different observations, use sticky notes, or just write in the boxes.

FIGURE 14 ● Example Note-Taking Observation Tool

Strategy: Break Apart to Multiply   Type of Numbers: two-digit × two-digit

R. Attis: Using rectangles and breaking apart in 4 parts	R. Barnes: Excellent at breaking apart one of the factors	G. Carmes: Needs an illustration
H. Moore: Doing a lot mentally and quickly	M. Franke: Has difficulty choosing how to break apart	M. Gillis: Is using flexible ways to break apart, not just place
C. Kruz: Solving mentally with ease	W. Munez:	P. Smith: Tends to break apart first number
M. Roberts:	M. Rodriguez: Solving quickly using some written steps	S. Sanchez: Always changes both numbers

online resources This resource can be downloaded at **resources.corwin.com/FOF/multiplydividewholenumber**.

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Some days, you collect data on some students; other days, you collect data on other students. The data can help in classroom discussions and in planning for instructional next steps.

Journal prompts provide an opportunity for students to write about their thinking process. Each module provides a collection of prompts that you might use for journaling. You can modify those or easily craft your own. The prompt can specifically ask students to explain how they used the strategy:

> **Explain and show how you can use Compensation to solve 6×498.**

Or a prompt can focus on identifying when that strategy is a good idea:

> **Circle the problems that are good choices for solving with the Compensation strategy and tell why you selected them:**
>
> 495×3 7×726 440×2 399×6

Interviewing is an excellent way to really understand student thinking. You can pick any problem that lends to the strategy you are working on and write it on a note card (or record two or three on separate notecards). While students are engaged in an instructional or practice activity, roam the room, select a child, show them a card, and ask them (1) to solve it and (2) explain how they thought about it. You can pair this with an observation tool to keep track of how each student is progressing.

HOW DO I USE PART 3 TO SUPPORT STUDENTS' FLUENCY?

As soon as students know more than one way, it is time to integrate routines, tasks, centers, and games that focus on choosing when to use a strategy. That is where Part 3 comes in. As you read in Fluency Action 1, students need to be able to choose efficient strategies. The strategy modules provide students *access* to those strategies, ensuring the strategies make sense and giving students ample opportunities to practice those strategies and become adept at using them. However, if you stop there, students are left on their own when it comes to choosing which strategy to use when. It is like having a set of knives in the kitchen but not knowing which ones to use for slicing cheese or bread, cutting meat, or chopping vegetables. Like with food, some items can be cut with various knives, but other food really needs a specific knife.

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Do not wait until after all strategies are learned to focus on when to use a strategy—instead weave in Part 3 activities regularly. Each time a new strategy is learned, it is time to revisit activities that engage students in making choices from among the strategies in their repertoire. Students must learn what to look for in a problem to decide which strategy they will use to solve the problem *efficiently* based on the numbers in the problem. This is *flexibility* in action, and thus leads to fluency.

TEACHING TAKEAWAY

Teaching for fluency means teaching strategies as core instruction, routinely practicing them, and offering opportunities for students to choose among strategies.

"FACTORS" IN GETTING THE BEST "PRODUCT"

Part 1 has briefly described factors that are important in developing fluency, and these ideas are important as you implement activities from the modules. We close Part 1 with five key factors to figuring out fluency.

1. Be clear on what fluency means (three components and six actions). This includes communicating it to students and their parents.
2. Attend to readiness skills: conceptual understanding, properties, utilities, computational estimation, and, of course, basic fact fluency.
3. Through activities and discussion, help students connect on the features of a problem and how that relates to good strategy options.
4. Reinforce student reasoning and choice selection, rather than focus on speed and accuracy. Getting the strategies down initially takes more time, but eventually will become more efficient.
5. Assess fluency, not just accuracy.

Time invested in strategy work has big payoffs—confident and fluent students (and that is the "best product"). That is why we have so many activities in this book. Teach the strategies as part of core instruction, *and* continue to practice throughout the year, looping back to strategies that students might be forgetting to use (with Part 2 activities) and offering ongoing opportunities to choose from among strategies (with Part 3 activities).

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