WHAT YOUR COLLEAGUES ARE SAYING . . .

"This text provides a clear definition of what fluency really is and provides strategies to deepen students' number sense and make them fluent mathematicians."

Meghan Schofield
Third-Grade Teacher

"I wish I'd had this book when I was in the classroom 10 years ago. The authors clearly lay out a pathway to procedural fluency, including intentional activities to understand and practice specific strategies, while also advocating for space for students to make decisions and feel empowered as mathematical thinkers and doers."

Kristine M. Gettelman
Instructional Designer
CenterPoint Education

"This book is a must-read for teachers wanting to learn more about focused math fluency instruction. The steps are clear and easy to follow. You will have all the steps to help your students become fluent math thinkers."

Carly Morales
Instructional Coach
District 93

"Are you ready to help your students connect their Number Talks and number routines to the real world? *Figuring Out Fluency* will give you the routines, games, protocols, and resources you need to help your students build their fluency in number sense (considering reasonableness, strategy selection, flexibility, and more). Our students deserve the opportunity to build a positive and confident mathematics identity. We can help support them to build this identity by providing them with access to a variety of strategies and the confidence to know when to use them."

Sarah Gat
Instructional Coach
Upper Grand District School Board

"*Figuring Out Fluency* goes beyond other resources currently on the market. It not only provides a robust collection of strategies and routines for developing fluency but also pays critical attention to the ways teachers can empower each and every student as a mathematical thinker who can make strategic decisions about their computation approaches. If you are looking for instruction and assessment approaches for fluency that move beyond getting the right answer, this is the resource for you."

Nicole Rigelman
Professor of Mathematics Education
Portland State University

"This book should be on every teacher's desk as a tool for building fluency. Many times, I teach a strategy and wonder why my students go back to a slower, less useful strategy. This book answers that question for me. The games give students a fun and engaging way to use a practice strategy. I can easily differentiate any game for a variety of learners."

Barb Klein
Third-Grade teacher

"*Figuring Out Fluency* provides a wealth of insightful examples and resources to support teachers, students, and parents in learning about and truly understanding computational fluency. As a math coach, I am excited to use this book to plan and teach meaningful lessons with teachers to model efficient strategies for students as they add and subtract."

Marcy Myers
Elementary Mathematics Instructional Leader
Carroll County Public Schools

"Being fluent is much more than solving problems quickly and accurately. This jam-packed resource brings clarity to what it means to be fluent with whole number addition and subtraction and provides numerous ideas for strategy instruction, purposeful practice, and assessment. It's an absolute must-have for everyone who strives to support students in reaching the goal of fluency!!"

Susie Katt
K–2 Mathematics Coordinator
Lincoln Public Schools

"This book is invaluable. SanGiovanni, Bay-Williams, and Serrano don't just provide top tier content for educators, they include strategy briefs for families! I absolutely love the idea of including parents as "partners in the pursuit of fluency.""

Cherelle C. McKnight
Mathematics Specialist, P.O.W.E.R. Academic Strength and Conditioning

"The term *math fluency* may be interpreted in different ways depending on who you ask. This book clearly defines fluency and is a must-have resource for all educators working toward this goal with their students. Each module focuses on different strategies and ways to implement using parent resources, centers, games, and teaching tips. This is the book that you can read and put into immediate action in your classroom."

Cindy Cliche
Murfreesboro City Schools
Math Coordinator

"For years research has indicated that fluency is much more than speed, yet timed assessments and traditional instruction persist for teachers without a clear vision or tools to change their practices. This series provides teachers with the explicit examples, resources, and activities needed to bring that research to life for their students and will quickly become a well-worn guidebook for every fluency-focused classroom. This is the toolkit teachers have been yearning for in their journey toward fluency with their students."

Gina Kilday
Elementary Math Interventionist and MTSS Coordinator

"Fluency isn't a dry landscape of disconnected facts, it is a rich soil for developing and connecting diverse perspectives and ideas. This book series equips you with a deep understanding of fluency and a variety of activities to engage students in co-constructing ideas about addition and subtraction that will last a lifetime."

Berkeley Everett
Math Coach and Facilitator for UCLA Mathematics Project
Math Consultant for DragonBox

"*Figuring Out Fluency* is a must-have, must-read, and must-use for elementary teachers. This text is concise and intentional in communicating the aspects of math fluency and what they look like in action. The routines, tasks, and activities included are easy to implement yet impactful and engaging. Teachers will gain insight for their pedagogy when reading, and students will make strides as problem solvers and flexible thinkers engaging in the activities."

Sumer Smith
Second Grade Teacher

"This book—indeed this *series*—is a must-read for elementary and middle level teachers, coaches, and administrators. Within this resource you will find a synthesis of important research organized to help readers develop a clear and common understanding of fluency paired with a large collection of teaching activities that provide concrete ways to support students' fluency development. *Figuring Out Fluency* provides a much-needed roadmap for teachers looking to increase computational proficiency with multiplication and division."

Delise Andrews
3–5 Mathematics Coordinator Lincoln Public Schools

"The authors John J. SanGiovanni, Jennifer M. Bay-Williams, and Rosalba Serrano shine a bright light on how math fluency is *the* equity issue in mathematics education. How refreshing to have a book that equips math educators with the research and strategies to make a difference for *all* students! Let's implement these strategy modules in this book and help kids figure out fluency once and for all!"

Kelly DeLong
Executive Director for the Kentucky Center for Mathematics
Northern Kentucky University

Figuring Out Fluency—Addition and Subtraction With Whole Numbers: A Classroom Companion

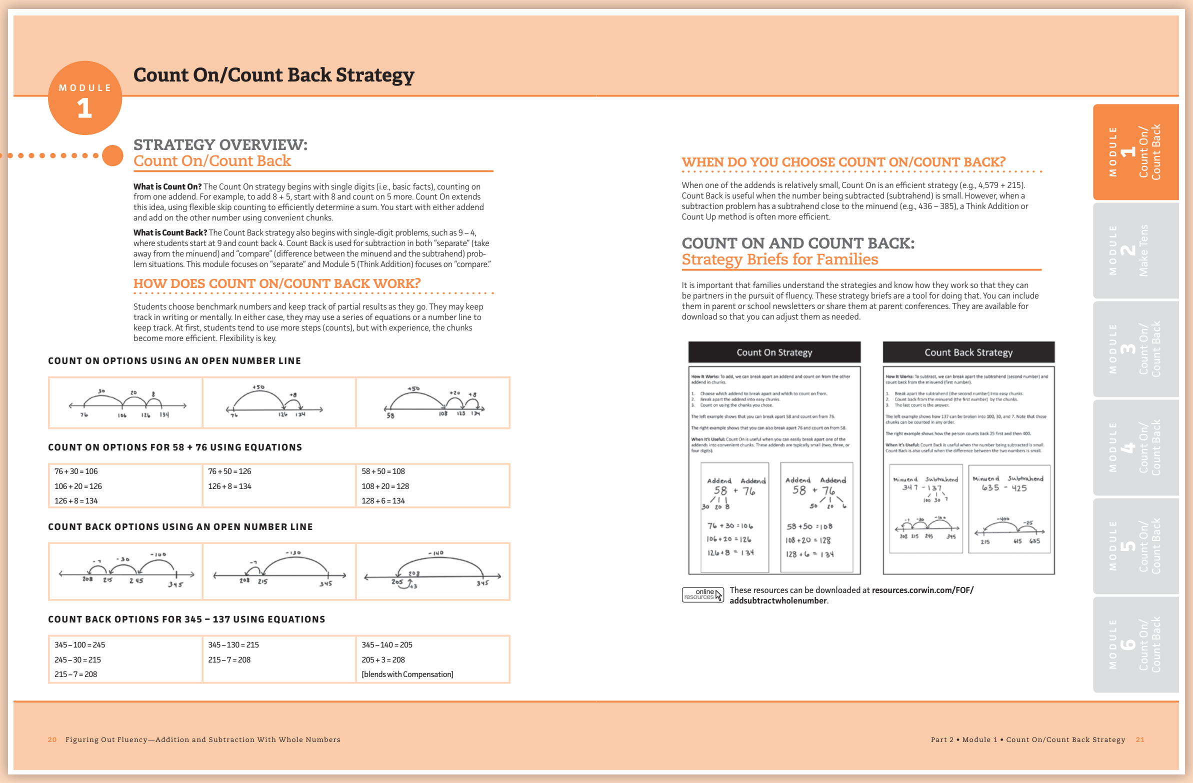
The Book at a Glance

Building off of *Figuring Out Fluency*, this classroom companion dives deep into five of the Seven Significant Strategies that relate to procedural fluency when adding and subtracting whole numbers, beyond basic facts.

FIGURE 12 ● Reasoning Strategies for Adding and Subtracting Whole Numbers

REASONING STRATEGIES	RELEVANT OPERATIONS
1. Count On/Count Back (Module 1)	Addition and Subtraction
2. Make Tens (Module 2)	Addition
3. Partial Sums and Differences (Module 3)	Addition and Subtraction
4. Compensation (Module 4)	Addition and Subtraction
5. Think Addition (Module 5)	Subtraction

Strategy overviews and family briefs communicate how each strategy helps students develop flexibility, efficiency, accuracy, automaticity, and reasonableness.



MODULE 1

Count On/Count Back Strategy

STRATEGY OVERVIEW: Count On/Count Back

What is Count On? The Count On strategy begins with single digits (i.e., basic facts), counting on from one addend. For example, to add 8 + 5, start with 8 and count on 5 more. Count On extends this idea, using flexible skip counting to efficiently determine a sum. You start with either addend and add on the other number using convenient chunks.

What is Count Back? The Count Back strategy also begins with single-digit problems, such as 9 – 4, where students start at 9 and count back 4. Count Back is used for subtraction in both "separate" (take away from the minuend) and "compare" (difference between the minuend and the subtrahend) problem situations. This module focuses on "separate" and Module 5 (Think Addition) focuses on "compare."

HOW DOES COUNT ON/COUNT BACK WORK?

Students choose benchmark numbers and keep track of partial results as they go. They may keep track in writing or mentally. In either case, they may use a series of equations or a number line to keep track. At first, students tend to use more steps (counts), but with experience, the chunks become more efficient. Flexibility is key.

COUNT ON OPTIONS USING AN OPEN NUMBER LINE

COUNT ON OPTIONS FOR 58 + 76 USING EQUATIONS

76 + 30 = 106 106 + 20 = 126 126 + 8 = 134	76 + 50 = 126 126 + 8 = 134	58 + 50 = 108 108 + 20 = 128 128 + 6 = 134

COUNT BACK OPTIONS USING AN OPEN NUMBER LINE

COUNT BACK OPTIONS FOR 345 – 137 USING EQUATIONS

345 – 100 = 245 245 – 30 = 215 215 – 7 = 208	345 – 130 = 215 215 – 7 = 208	345 – 140 = 205 205 + 3 = 208 [blends with Compensation]

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WHEN DO YOU CHOOSE COUNT ON/COUNT BACK?

When one of the addends is relatively small, Count On is an efficient strategy (e.g., 4,579 + 215). Count Back is useful when the number being subtracted (subtrahend) is small. However, when a subtraction problem has a subtrahend close to the minuend (e.g., 436 – 385), a Think Addition or Count Up method is often more efficient.

COUNT ON AND COUNT BACK: Strategy Briefs for Families

It is important that families understand the strategies and know how they work so that they can be partners in the pursuit of fluency. These strategy briefs are a tool for doing that. You can include them in parent or school newsletters or share them at parent conferences. They are available for download so that you can adjust them as needed.

Count On Strategy

How It Works: To add, we can break apart an addend and count on from the other addend in chunks.

1. Choose which addend to break apart and which to count on from.
2. Break apart the addend into easy chunks.
3. Count on using the chunks you chose.

The left example shows that you can break apart 58 and count on from 76.

The right example shows that you can also break apart 76 and count on from 58.

When It's Useful: Count On is useful when you can easily break apart one of the addends into convenient chunks. These addends are typically small (two, three, or four digits).

Count Back Strategy

How It Works: To subtract, we can break apart the subtrahend (second number) and count back from the minuend (first number).

1. Break apart the subtrahend (the second number) into easy chunks.
2. Count back from the minuend (the first number) by the chunks.
3. The last count is the answer.

The left example shows how 137 can be broken into 100, 30, and 7. Note that those chunks can be counted in any order.

The right example shows how the person counts back 25 first and then 400.

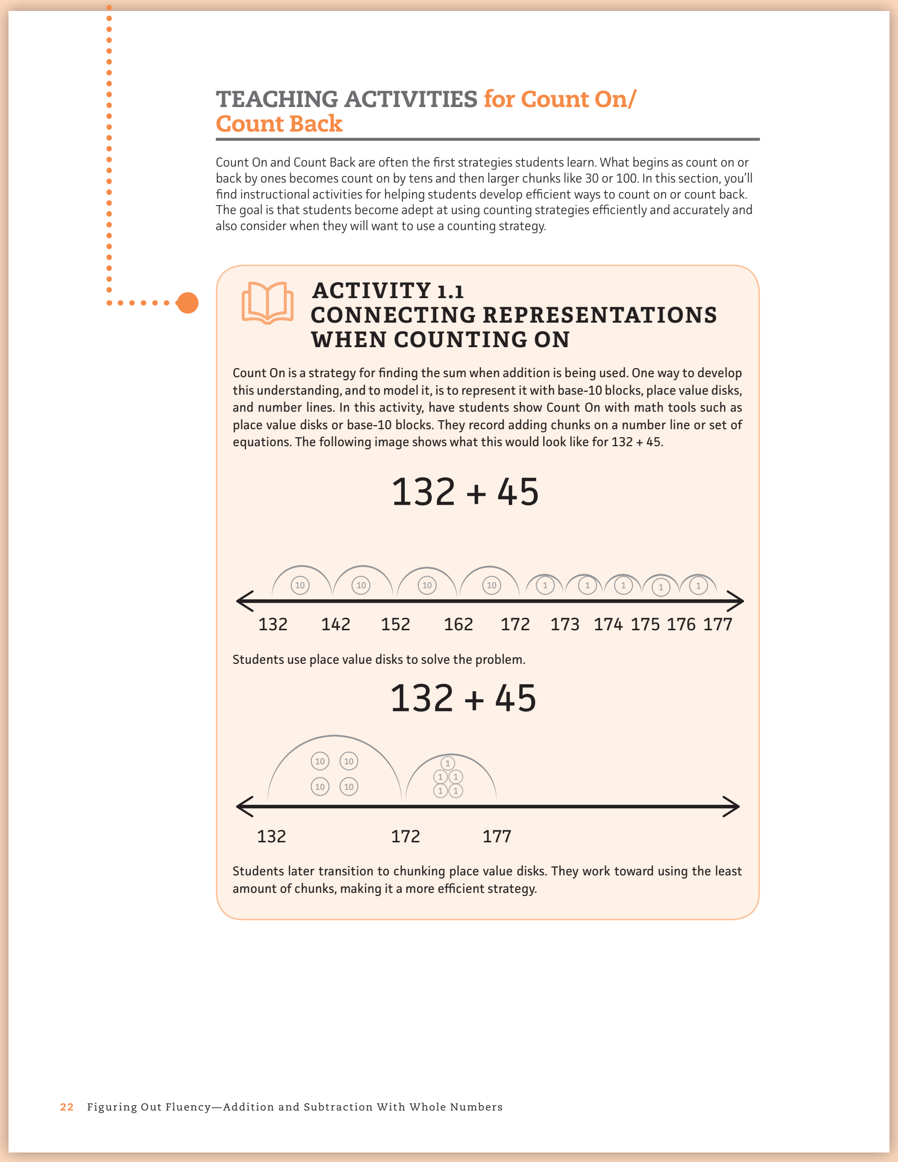
When It's Useful: Count Back is useful when the number being subtracted is small. Count Back is also useful when the difference between the two numbers is small.

online resources These resources can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

MODULE 1 Count On/Count Back
MODULE 2 Make Tens
MODULE 3 Count On/Count Back
MODULE 4 Count On/Count Back
MODULE 5 Count On/Count Back
MODULE 6 Count On/Count Back

Part 2 • Module 1 • Count On/Count Back Strategy 21

Each strategy module starts with teaching activities that help you explicitly teach the strategy.



TEACHING ACTIVITIES for Count On/ Count Back

Count On and Count Back are often the first strategies students learn. What begins as count on or back by ones becomes count on by tens and then larger chunks like 30 or 100. In this section, you'll find instructional activities for helping students develop efficient ways to count on or count back. The goal is that students become adept at using counting strategies efficiently and accurately and also consider when they will want to use a counting strategy.

ACTIVITY 1.1 CONNECTING REPRESENTATIONS WHEN COUNTING ON

Count On is a strategy for finding the sum when addition is being used. One way to develop this understanding, and to model it, is to represent it with base-10 blocks, place value disks, and number lines. In this activity, have students show Count On with math tools such as place value disks or base-10 blocks. They record adding chunks on a number line or set of equations. The following image shows what this would look like for 132 + 45.

132 + 45

10 10 10 10 1 1 1 1 1

132 142 152 162 172 173 174 175 176 177

Students use place value disks to solve the problem.

132 + 45

10 10 10 10 1 1 1 1 1

132 172 177

Students later transition to chunking place value disks. They work toward using the least amount of chunks, making it a more efficient strategy.

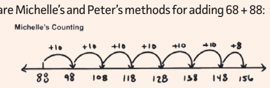
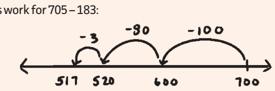
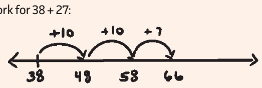
SanGiovanni FoF_Add_Subt.indb 5 7/28/21 8:03 AM

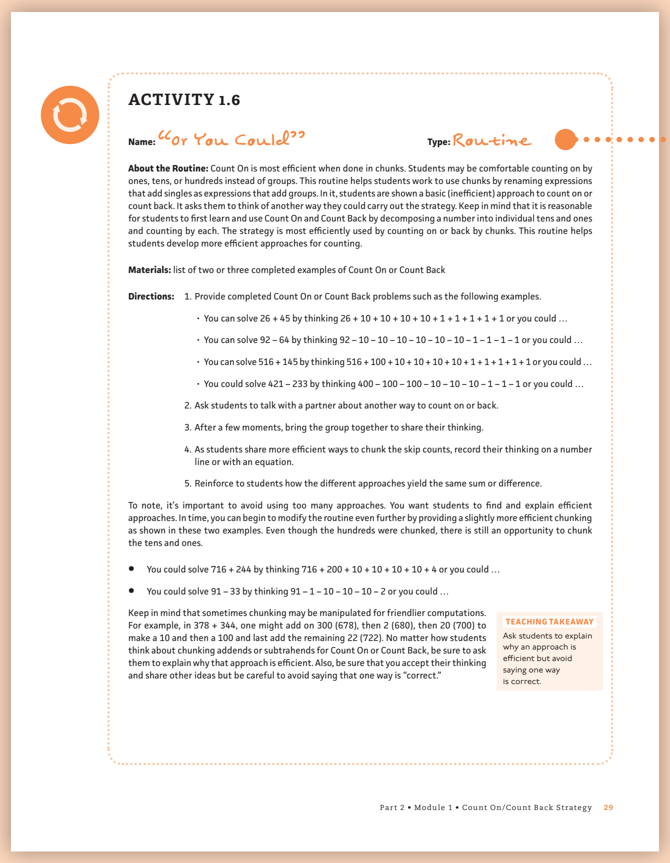
Each strategy shares worked examples for you to work through with your students as they develop their procedural fluency.

2. The student makes an error in breaking apart a number.
   - 87 + 71: from 87, counts on 20 to get to 107 and then 40 [rather than 50] to get to 147 and one more to 148.
   - 4,225 – 1,270: counts back 1,200 to 3,025, then back 25 to 3,000, then back 55 [rather than 45] to 2,945.
3. The student misses a count when consecutive digits are the same.
   - 871 – 449: counts back 4 hundreds to 471 but overlooks 4 tens because a jump of 4 was just made.

Although not an error, students may continue to count by single ones, tens, or hundreds instead of counting in chunks or multiples of these. Show a worked example that does this and ask students to consider how they might combine jumps to add/subtract more efficiently. Various worked examples can be found throughout this module, or you can collect authentic worked examples (see prompts from Activity 1.5, for example). Additionally, have students compare worked examples to highlight different ways to count on or back. A sampling of ideas are provided in the following table.

SAMPLE WORKED EXAMPLES FOR COUNT ON/ COUNT BACK

Correctly Worked Example (make sense of the strategy) What did _____ do? Why does it work? Is this a good method for this problem?	Compare Michelle's and Peter's methods for adding 68 + 88:  	
Partially Worked Example (implement the strategy accurately) Why did _____ start the problem this way? What does _____ need to do to finish the problem?	Nate's start for 546 – 175: 546 - 200 = 346	Jeannette's start for 546 – 175: 546 - 140 = 406
Incorrectly Worked Example (highlight common errors) What did _____ do? What mistake does _____ make? How can this mistake be fixed?	Luke's work for 705 – 183:  Allie's work for 38 + 27: 	



ACTIVITY 1.6

Name: "Or You Could" **Type:** Routine

About the Routine: Count On is most efficient when done in chunks. Students may be comfortable counting on by ones, tens, or hundreds instead of groups. This routine helps students work to use chunks by renaming expressions that add singles as expressions that add groups. In it, students are shown a basic (inefficient) approach to count on or count back. It asks them to think of another way they could carry out the strategy. Keep in mind that it is reasonable for students to first learn and use Count On and Count Back by decomposing a number into individual tens and ones and counting by each. The strategy is most efficiently used by counting on or back by chunks. This routine helps students develop more efficient approaches for counting.

Materials: list of two or three completed examples of Count On or Count Back

Directions:

1. Provide completed Count On or Count Back problems such as the following examples.
   - You can solve 26 + 45 by thinking 26 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 or you could …
   - You can solve 92 – 64 by thinking 92 – 10 – 10 – 10 – 10 – 10 – 10 – 1 – 1 – 1 – 1 or you could …
   - You can solve 516 + 145 by thinking 516 + 100 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 or you could …
   - You could solve 421 – 233 by thinking 400 – 100 – 100 – 10 – 10 – 10 – 1 – 1 – 1 or you could …
2. Ask students to talk with a partner about another way to count on or back.
3. After a few moments, bring the group together to share their thinking.
4. As students share more efficient ways to chunk the skip counts, record their thinking on a number line or with an equation.
5. Reinforce to students how the different approaches yield the same sum or difference.

To note, it's important to avoid using too many approaches. You want students to find and explain efficient approaches. In time, you can begin to modify the routine even further by providing a slightly more efficient chunking as shown in these two examples. Even though the hundreds were chunked, there is still an opportunity to chunk the tens and ones.

- You could solve 716 + 244 by thinking 716 + 200 + 10 + 10 + 10 + 10 + 4 or you could …
- You could solve 91 – 33 by thinking 91 – 1 – 10 – 10 – 10 – 2 or you could …

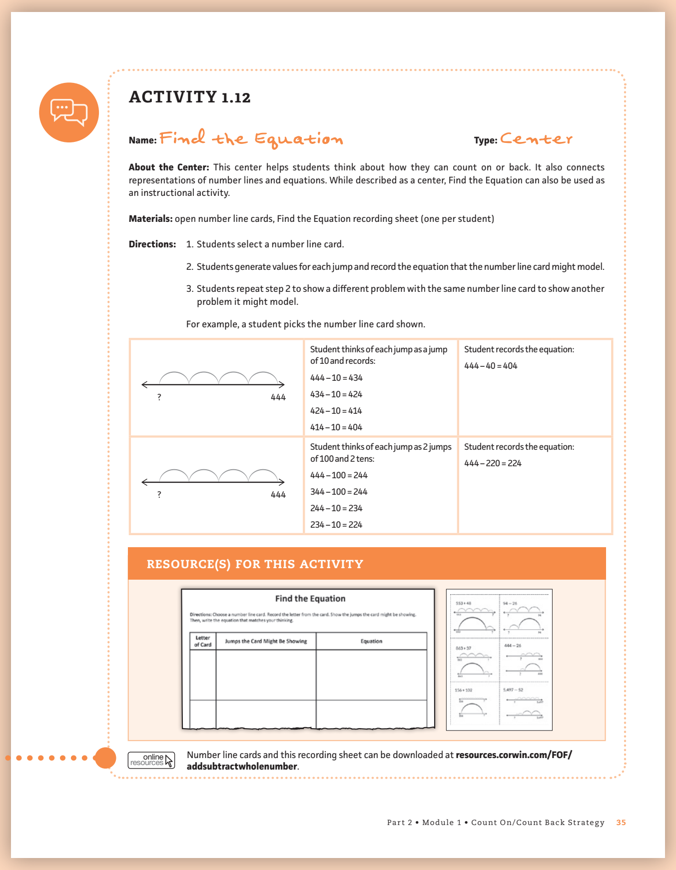
Keep in mind that sometimes chunking may be manipulated for friendlier computations. For example, in 378 + 344, one might add on 300 (678), then 2 (680), then 20 (700) to make a 10 and then a 100 and last add the remaining 22 (722). No matter how students think about chunking addends or subtrahends for Count On or Count Back, be sure to ask them to explain why that approach is efficient. Also, be sure that you accept their thinking and share other ideas but be careful to avoid saying that one way is "correct."

TEACHING TAKEAWAY
Ask students to explain why an approach is efficient but avoid saying one way is correct.

Part 2 • Module 1 • Count On/Count Back Strategy 29

Routines, Games, and Centers for each strategy offer extensive opportunity for student practice.

Download the resources you need for each activity at this book's companion website.



ACTIVITY 1.12

Name: Find the Equation **Type:** Center

About the Center: This center helps students think about how they can count on or back. It also connects representations of number lines and equations. While described as a center, Find the Equation can also be used as an instructional activity.

Materials: open number line cards, Find the Equation recording sheet (one per student)

Directions:

1. Students select a number line card.
2. Students generate values for each jump and record the equation that the number line card might model.
3. Students repeat step 2 to show a different problem with the same number line card to show another problem it might model.

For example, a student picks the number line card shown.

? 444	Student thinks of each jump as a jump of 10 and records: 444 – 10 = 434 434 – 10 = 424 424 – 10 = 414 414 – 10 = 404	Student records the equation: 444 – 40 = 404
? 444	Student thinks of each jump as 2 jumps of 100 and 2 tens: 444 – 100 = 244 344 – 100 = 244 244 – 10 = 234 234 – 10 = 224	Student records the equation: 444 – 220 = 224

RESOURCE(S) FOR THIS ACTIVITY

Find the Equation

Letter of Card	Jumps the Card Might Be Showing	Equation

553 + 48
94 – 26
863 + 37
444 – 26
156 + 102
5,497 – 52

online resources Number line cards and this recording sheet can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

Part 2 • Module 1 • Count On/Count Back Strategy 35

FIGURING OUT Fluency
ADDITION & SUBTRACTION
With Whole Numbers

Grades K–8

FIGURING OUT Fluency

ADDITION & SUBTRACTION With Whole Numbers

Grades K–8

A Classroom Companion

John J. SanGiovanni
Jennifer M. Bay-Williams
Rosalba Serrano

CORWIN Mathematics

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For information:

Corwin
A SAGE Company
2455 Teller Road
Thousand Oaks, California 91320
(800) 233–9936
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President: Mike Soules
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Copy Editor: Christina West
Typesetter: Integra
Proofreader: Susan Schon
Indexer: Integra
Cover Designer: Rose Storey
Marketing Manager: Margaret O'Connor

Printed in the United States of America.

Library of Congress Cataloging-in-Publication Data

Names: SanGiovanni, John, author. | Bay-Williams, Jennifer M., author. | Serrano, Rosalba, author.
Title: Figuring out fluency - addition and subtraction with whole numbers : a classroom companion / John J. SanGiovanni, Jennifer M. Bay-Williams, Rosalba Serrano.
Description: Thousand Oaks, California : Corwin Press, Inc., [2022] | Includes bibliographical references and index.
Identifiers: LCCN 2021019510 (print) | LCCN 2021019511 (ebook) | ISBN 9781071825099 (paperback) | ISBN 9781071851593 | ISBN 9781071851586
Subjects: LCSH: Arithmetic--Study and teaching (Elementary) | Arithmetic--Study and teaching (Middle school) | Numbers, Natural. | Mathematical fluency.
Classification: LCC QA135.6 .S2577 2022 (print) | LCC QA135.6 (ebook) | DDC 372.7/2--dc23
LC record available at https://lccn.loc.gov/2021019510
LC ebook record available at https://lccn.loc.gov/2021019511

This book is printed on acid-free paper.

21 22 23 24 25 10 9 8 7 6 5 4 3 2 1

Contents

online resources

Visit the companion website at **resources.corwin.com/FOF/addsubtractwholenumber** for downloadable resources.

Preface

Fluency is an equity issue. In written documents and in our daily work, we (mathematics teachers and leaders) communicate that every student must be *fluent* with whole number addition and subtraction, for example. But we haven't even come close to accomplishing this for each and every student. The most recent National Assessment of Educational Progress (NAEP) data, for example, finds that about two-fifths (41%) of the nation's Grade 4 students are at or above proficient and about one-third (34%) of our nation's Grade 8 students at or above proficient (NCES, 2019). We can and must do better! One major reason we haven't been able to develop fluent students is that there are misunderstandings about what fluency really means.

FIGURING OUT FLUENCY

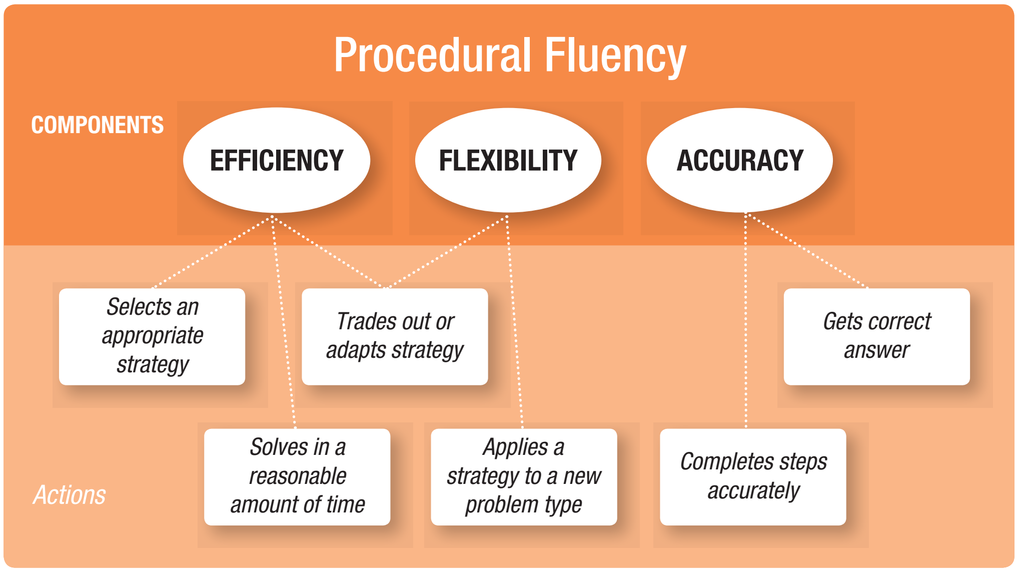
In order to ensure every student develops fluency, we first must:

- Understand what procedural fluency is (and what it isn't),
- Respect fluency, and
- Plan to explicitly teach and assess reasoning strategies.

If you have read our anchor book *Figuring Out Fluency for Mathematics Teaching and Learning*—which we recommend in order to get the most out of *this* classroom companion—you'll remember an in-depth discussion of these topics.

WHAT PROCEDURAL FLUENCY IS AND ISN'T

Like fluency with language, wherein you decide how you want to communicate an idea, fluency in mathematics involves decision-making as you decide how to solve a problem. In our anchor book, we propose the following visual as a way to illustrate the full meaning of fluency.



Source: Adapted with permission from D. Spangler & J. Wanko (Eds.), *Enhancing Classroom Practice with Research behind Principles to Actions*, copyright 2017, by the National Council of Teachers of Mathematics. All rights reserved.

Part 1 of this book explains the elements of this procedural fluency graphic. To cut to the chase, procedural fluency is much more than knowing facts and standard algorithms. Fluency involves higher-level thinking, wherein a person analyzes a problem, considers options for how to solve it, selects an efficient strategy, and accurately enacts that strategy (trading it out for another if it doesn't go well). Decision-making is key and that means you need to have good options to choose from. This book provides instructional and practice activities so that students learn different options (Part 2) and then provides practice activities to help students learn to choose options (Part 3).

RESPECT FLUENCY

We are strong advocates for conceptual understanding. We all must be. But there is not a choice here. Fluency relies on conceptual understanding, and conceptual understanding alone cannot help students fluently navigate computational situations. They go together and must be connected. Instructional activities throughout Part 2 provide opportunities for students to discuss, critique, and justify their thinking, connecting their conceptual understanding to their procedural knowledge and vice versa.

EXPLICITLY TEACH AND ASSESS REASONING STRATEGIES

If every student is to be fluent in whole number addition and subtraction, then every student needs access to the significant strategies for these operations. And there must be opportunities for students to learn how to select the best strategy for a particular problem. For example, students may learn that the Make Hundreds strategy works well when one or both of the addends is close to a hundred (e.g., 495 + 637) but not as well when neither addend is close to a hundred (e.g., 458 + 371). To accomplish this, all three fluency components must have equitable attention in instruction and assessment. This is a major shift from traditional teaching and assessing, which privileges accuracy over the other two components and the standard algorithm over reasoning strategies.

Let's unpack the phrase *explicit strategy instruction*. According to the *Merriam-Webster Dictionary*, *explicit* means "fully revealed or expressed without vagueness" ("Explicit," 2021). In mathematics teaching, being *explicit* means making mathematical relationships visible. A strategy is a flexible method to solve a problem. *Explicit strategy instruction*, then, is engaging students in ways to clearly see how and why a strategy works. For example, with addition, students might compare 100 + 15 and 98 + 17 on the number line as a way to reveal how the Compensation strategy works. With subtraction, students might be asked to explain why 75 – 58 = 77 – 60 using Unifix Cubes, tiny ten-frame cards, or a number line. Proving examples leads to a generalization that you can change both values of a subtraction expression by the same amount, and the answer will be the same (compensation!). Once understood, students need to explore *when* a strategy is a good option. Learning how to use and how to choose strategies *empowers* students to be able to decide how they want to solve a problem, developing a positive mathematics identity and a sense of agency.

USING THIS BOOK

This book is a classroom companion to *Figuring Out Fluency for Mathematics Teaching and Learning*. In that anchor book, we lay out what fluency is, identify the fallacies that stand in the way of a true focus on fluency, and elaborate on necessary foundations for fluency. We also propose the following:

- Seven Significant Strategies across the operations, five of which apply to whole number addition and subtraction
- Eight "automaticies" *beyond* automaticity with basic facts, four of which are relevant to whole number addition and subtraction
- Five ways to engage students in meaningful practice
- Four assessment options that can replace (or at least complement) tests and that focus on real fluency
- Many ways to engage families in supporting their child's fluency

In Part 1 of this book, we revisit some of these ideas in order to connect specifically to whole number addition and subtraction. This section is not a substitute for the anchor book, but rather a brief revisiting of central ideas that serve as reminders of what was fully illustrated, explained, and justified in *Figuring Out Fluency in Mathematics Teaching and Learning*. Hopefully, you have had the chance to read and engage with that content with colleagues *first*, and then Part 1 will help you think about those ideas as they apply to whole number addition and subtraction. Finally, Part 1 includes suggestions for how to use the strategy modules.

Part 2 is focused on explicit instruction of each significant strategy for whole number addition and subtraction. Each module includes the following:

- An overview for your reference and to share with students and colleagues
- A strategy brief for families
- A series of instructional activities, with the final one offering a series of questions to promote discourse about the strategy
- A series of practice activities, including worked examples, routines, games, and center activities that engage students in meaningful and ongoing practice to develop proficiency with that strategy

As you are teaching and find your students are ready to learn a particular strategy, pull this book off the shelf, go to the related module, and access the activities and ready-to-use resources. While the modules are sequenced in a developmental order overall, the order and focus on each strategy may vary depending on your grade and your students' experiences. Additionally, teaching within or across modules does not happen all at once; rather, activities can be woven into your instruction regularly, over time.

Part 3 is about becoming truly fluent—developing flexibility and efficiency with addition and subtraction of whole numbers. Filled with more routines, games, and centers, the focus here is on students *choosing* to use the strategies that make sense to them in a given situation. Part 3 also provides assessment tools to monitor students' fluency. As you are teaching and find your students are needing opportunities to choose from among the strategies they are learning, pull this book off the shelf and select an activity from Part 3.

In the Appendix, you will find lists of all the activities in order to help you easily locate what you are looking for by strategy or by type of resource.

This book can be used to complement or supplement any published mathematics program or district-created program. As we noted earlier, elementary mathematics has tended to fall short in its attention to efficiency and flexibility (and the related Fluency Actions illustrated in the earlier graphic). This book provides a large collection of activities to address these neglected components of fluency. Note that this book is part of a series that explores other operations and other numbers. You may also be interested in *Figuring Out Fluency for Whole Number Multiplication and Division* as well as the classroom companion books for decimal and fraction operations.

WHO IS THIS BOOK FOR?

With nearly 100 activities and a companion website with resources ready to download, this book is designed to support classroom teachers as they advance their students' fluency with whole number addition and subtraction. Special education teachers will find the explicit strategy instruction, as well as the additional practice, useful in supporting their students. Mathematics coaches and specialists can use this book for professional learning and to provide instructional resources to the classroom teachers they support. Mathematics supervisors and curriculum leads can use this book to help them assess fluency aspects of their mathematics curriculum and fill potential gaps in resources and understanding. Teacher preparation programs can use this book to galvanize preservice teachers' understanding of fluency and provide teacher candidates with a wealth of classroom-ready resources to use during internships and as they begin their career.

Acknowledgments

Just as there are many components to fluency, there are certainly many components to having a book like this come to fruition. The first component is the researchers and advocates who have defined procedural fluency and effective practices that support it. Research on student learning is hard work, as is defining effective teaching practices, and so we want to begin by acknowledging this work. We have learned from these scholars, and we ground our ideas in their findings. It is on their shoulders that we stand. Second are the teachers and their students who have taken up "real" fluency practices and shared their experiences with us. We would not have taken on this book had we not seen firsthand how a focus on procedural fluency in classrooms truly transforms students' learning and shapes their mathematics identities. It is truly inspiring! Additionally, the testimonies from many teachers about their own learning experiences as students and as teachers helped crystalize for us the facts and fallacies in this book. A third component to bringing this book to fruition was the family support to allow us to actually do the work. We are all grateful to our family members—expressed in our personal statements that follow—who supported us 24/7 as we wrote during a pandemic.

From Jennifer: I am forever grateful to my husband, Mitch, who is supportive and helpful in every way. I also thank my children, MacKenna and Nicolas, who often offer reactions and also endure a lot of talk about mathematics, including hearing every person at a family reunion talk about how they would solve 48 + 49. And that leads to my gratitude to my extended family—a mother who served on the school board for 13 years and helped me make it to a second year of teaching and a father who was a statistician and leader and helped me realize I could do "uncomfortable" things. My siblings—an accountant, a high school math teacher, and a university statistician—and their children have all supported my work on this book and the others in this series.

From John: I want to thank my family—especially my wife—who, as always, endure and support the ups and downs of taking on a new project. Thank you to Jenny and Rosalba for being exceptional partners. And thank you for dealing with my random thoughts, tangent conversations, and fantastic humor. As always, a heartfelt thank you to certain math friends and mentors for opportunities, faith in me, and support over the years. And thank you to my own math teachers who let me do math "my way," even if it wasn't "the way" back then.

From Rosalba: I want to thank my three boys, Daniel, Dylan, and Declan. All of the long days and hard work I do is to make the world a better place for you. Thank you for being so patient with your Madre and listening to countless math conversations. To John and Jenny, I cannot express enough thanks. Thank you for your advice, support, and encouragement throughout this process. I am forever grateful for this learning experience and the opportunity you have given me.

A fourth component is vision and writing support. We are so grateful to Corwin for recognizing the importance of defining and implementing procedural fluency in the mathematics classroom. Our editor and publisher, Erin Null, has gone above and beyond as a partner in the work, ensuring that our ideas are as well stated and useful as possible. The entire editing team at Corwin has been creative, thorough, helpful, and supportive.

As with fluency, no component is more important than another, and without any component, there is no book, so to the researchers, teachers, family, and editing team, thank you. We are so grateful.

PUBLISHER'S ACKNOWLEDGMENTS

Corwin gratefully acknowledges the contributions of the following reviewers:

Becky Evans
Grades 3–5 Math Teacher Leader
Lincoln Public Schools

Jamie Fraser
Math Consultant
Developer and Educational Distributer
Bound2Learn

Sarah Gat
Instructional Coach
Upper Grand District School Board

Kristine M. Gettelman
Instructional Designer
CenterPoint Education

Christina Hawley
Grade 1 and Grade 2 Teacher

Cathy Martin
Associate Chief of Academics
Denver Public Schools

Carly Morales
Instructional Coach
District 93

Margie Pearse
Math Coach

Nicole Rigelman
Professor of Mathematics Education
Portland State University

Meghan Schofield
Third-Grade Teacher

About the Authors

John J. SanGiovanni is a mathematics supervisor in Howard County, Maryland. There, he leads mathematics curriculum development, digital learning, assessment, and professional development. John is an adjunct professor and coordinator of the Elementary Mathematics Instructional Leadership graduate program at McDaniel College. He is an author and national mathematics curriculum and professional learning consultant. John is a frequent speaker at national conferences and institutes. He is active in state and national professional organizations, recently serving on the board of directors for the National Council of Teachers of Mathematics (NCTM) and currently as the president of the Maryland Council of Supervisors of Mathematics.

Jennifer M. Bay-Williams is a professor of mathematics education at the University of Louisville, where she teaches preservice teachers, emerging elementary mathematics specialists, and doctoral students in mathematics education. She has authored numerous books as well as many journal articles, many of which focus on procedural fluency (and other aspects of effective mathematics teaching and learning). Jennifer is a frequent presenter at national and state conferences and works with schools and districts around the world. Her national leadership includes having served as a member of the NCTM Board of Directors, on the TODOS: Mathematics for All Board of Directors, and as president and secretary of the Association of Mathematics Teacher Educators (AMTE).

Rosalba Serrano is an elementary mathematics consultant in New York and is the founder of Zenned Math, where she provides online professional development and coaching for elementary mathematics teachers. Rosalba has used her experience as a classroom teacher and mathematics coach to support teachers in deepening their understanding of mathematics and their use of effective teaching practices. A frequent speaker at both regional and national conferences, Rosalba is also active in mathematics organizations such as the NCTM, where she contributes to multiple committees. She has also worked as a professional development facilitator for various mathematics organizations and consults as a math editor for a number of publishing companies.

PART 1

FIGURING OUT FLUENCY

Key Ideas

WHAT IS FLUENCY WITH WHOLE NUMBER ADDITION AND SUBTRACTION?

To set the stage for figuring out fluency for whole number addition and subtraction, take a moment to do some math. Find the solution to each of these.

58 + 35	584 – 262	479 + 241
363 – 180	1,499 + 707	1,400 – 1,268

How did you find the sums and differences? Did you use the same approach or strategy for each? Did you move between different strategies? Did you change out a strategy based on the numbers within the problem? Did you start with one strategy and shift to another? You likely said "yes" to all of these questions because you fluently add and subtract whole numbers. Yet another reader can answer "yes" to each of these questions as well but solve each problem differently. This is true because fluency is a way of thinking rather than a way of doing. Thinking is unique to each individual. Thinking is grounded in parameters but its execution is left to the understanding, preference, and creativity of each thinker.

Of course, there are strategies used most frequently for certain problems (based on the numbers in the problem), but even in those examples, efficient alternatives are likely. For example, 1,499 + 707 may seem like a problem that "fits" Compensation: change 1,499 to 1,500 (add 1), add 1,500 + 707 (2,207), and adjust (subtract 1) to get 2,206 (Module 4). But a Partial Sums approach may "jump out" to another person who sees the problem this way: 1,400 + 700 (2,100) and 99 + 7 (106), combined is 2,206 (Module 3). Which method is closer to your way of thinking? Or did you think about it differently?

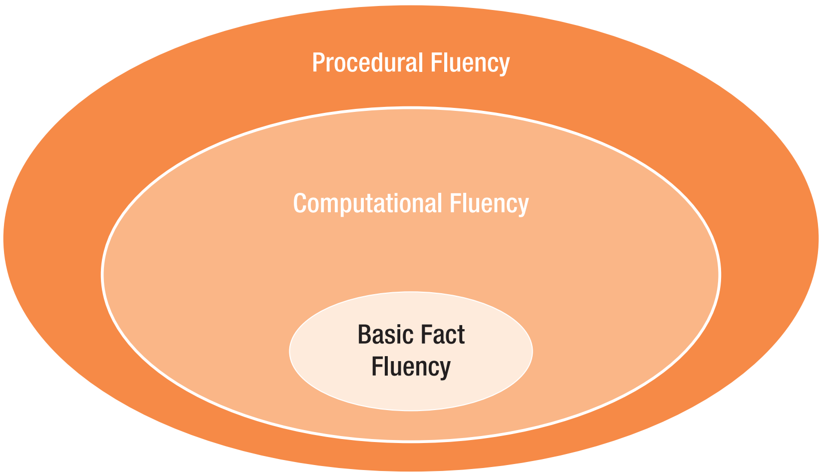
TEACHING TAKEAWAY

Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy.

Real fluency is the ability to select efficient strategies; to adapt, modify, or change out strategies; and to find solutions with accuracy. Real fluency is not the act of replicating someone else's steps or procedures for doing mathematics. It is the act of thinking, reasoning, and doing mathematics on one's own. Before fluency can be taught well, you must understand what fluency is and why it matters.

Procedural fluency is an umbrella term that includes basic fact fluency and computational fluency (see Figure 1). *Basic fact fluency* attends to fluently adding,

FIGURE 1 ● The Relationship of Different Fluency Terms in Mathematics



SanGiovanni FoF_Add_Subt.indb 2 7/28/21 8:03 AM

subtracting, multiplying, and dividing single-digit numbers. *Computational fluency* refers to the fluency in four operations across number types (whole numbers, fractions, etc.), regardless of the magnitude of the number. Procedural fluency encompasses both basic fact fluency and computational fluency, plus other procedures like finding equivalent fractions.

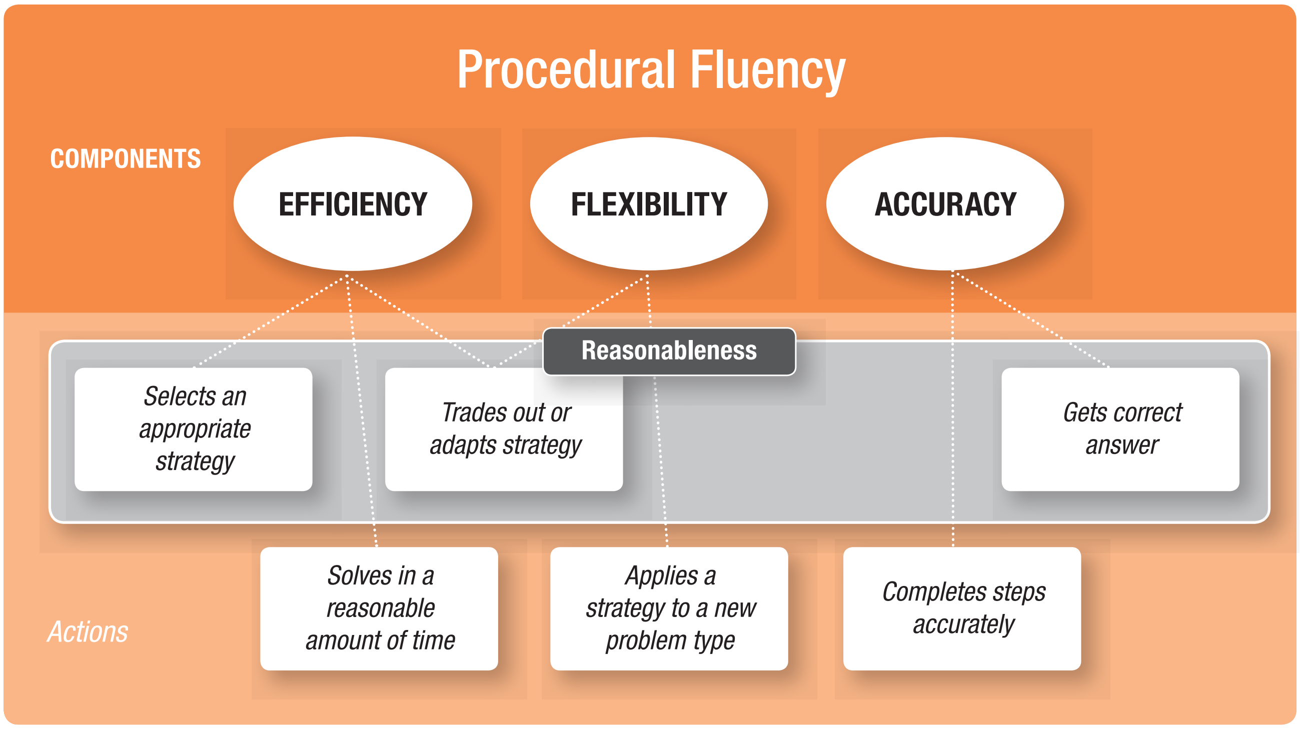
Beyond being an umbrella term that encompasses basic fact and computational fluency, procedural fluency is well defined as solving procedures efficiently, flexibly, and accurately (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics, 2014). These three **components** are defined as follows:

> *Efficiency:* solving a procedure in a reasonable amount of time by selecting an appropriate strategy and readily implementing that strategy
>
> *Flexibility:* knowing multiple procedures and can apply or adapt strategies to solve procedural problems (Baroody & Dowker, 2003; Star, 2005)
>
> *Accuracy:* correctly solving a procedure

Strategies are not the same as algorithms. Strategies are general methods that are flexible in design, compared to algorithms that are established steps implemented the same way across problems.

To focus on fluency, we need specific observable actions that we can look for in what students are doing in order to ensure they are developing fluency. We have identified six such actions. The three components and six Fluency Actions, and their relationships, are illustrated in Figure 2.

FIGURE 2 ● Procedural Fluency Components, Actions, and Checks for Reasonableness



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Three of the six Fluency Actions (should) attend to reasonableness. Fluency Actions and reasonableness are described later in Part 1, but first, it is important to consider why this "bigger" (comprehensive) view of fluency matters.

WHY FOCUS ON FLUENCY FOR WHOLE NUMBER ADDITION AND SUBTRACTION?

There are two key reasons why it's important to focus on fluency with whole number addition and subtraction. First, it is a critical foundation for ensuring that students fully realize procedural fluency in general, with all kinds of numbers. Fluency with whole numbers begins with developing fluency with single-digit addition and subtraction (the basic facts) and seeing how strategies such as Making 10 can be transferred to making a 30 or a 200 in order to add whole numbers efficiently (we call this generalized strategy Make Tens; basic facts are addressed in more detail later in Part 1). Fluency with addition and subtraction is a *necessity* for developing fluency with multiplication and division. For example, an efficient way to solve 9 × 35 is to think 10 × 35 (350) and subtract a group of 35 (315). This final step (350 – 35) requires subtraction fluency. A student needs to have a quick—often mental—way to do this subtraction, perhaps Counting Back, subtracting 30, and then subtracting 5. Fluency with whole number addition and subtraction is also a necessary foundation for fluency with rational numbers. Students will adapt their reasoning strategies with whole numbers to become fluent in adding and subtracting fractions and decimals (e.g., adapting Make Tens to Make a Whole). These connections are a focus of our anchor book, *Figuring Out Fluency in Mathematics Teaching and Learning* (hereafter titled *Figuring Out Fluency*).

Second and most importantly, developing fluency is an equity issue. Equipping students with options for how to solve addition and subtraction problems and positioning students to choose a method that works best for them develops a positive mathematics identity and sense of agency. Conversely, trying to remember algorithms and feeling anxiety about being correct or fast develops a negative mathematics identity and lack of agency.

WHAT DO FLUENCY ACTIONS LOOK LIKE FOR WHOLE NUMBER ADDITION AND SUBTRACTION?

The six Fluency Actions are observable and therefore form a foundation for assessing student progress toward fluency. Let's take a look at what each of these actions looks like in the context of whole number addition and subtraction.

FLUENCY ACTION 1: Select an Appropriate Strategy

TEACHING TAKEAWAY

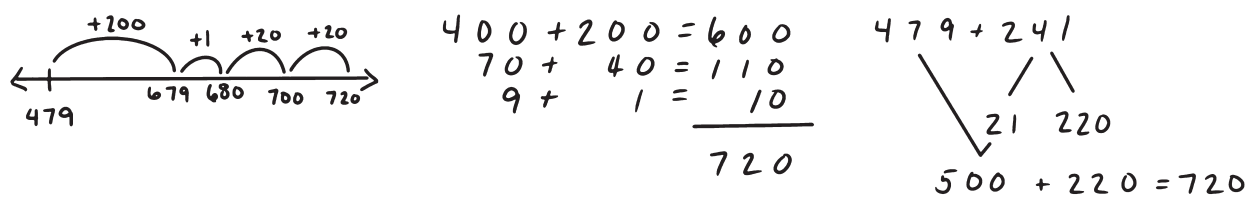
Selecting *an* appropriate strategy does not mean selecting *the* appropriate strategy. Many problems can be solved efficiently in more than one way.

Selecting *an* appropriate strategy does not mean selecting *the* appropriate strategy. Many problems can be solved efficiently in more than one way. Here is our operational definition: Of the available strategies, the one the student opts to use gets to a solution in about as many steps and/or about as much time as other appropriate options.

Consider 479 + 241. One could Count On from 479 (Figure 3a), Use Partial Sums (Figure 3b), or Make Hundreds (Figure 3c). Each of these strategies is appropriate for this problem but may not be as appropriate for other problems such as 500 + 363 or 299 + 476. Notice that this Fluency Action is one of three connected to *reasonableness*. Within this action is noticing when a strategy "fits" the numbers in the problem.

FIGURE 3 ● Adding 479 + 241

a. Count On b. Partial Sums c. Make Hundreds



A strategy cannot be used until it is understood. Once understood, a strategy becomes part of a student's repertoire of options and they are then able to select the strategy.

Importantly, we name strategies so that we can talk about them. But one approach may fit within various types of strategies and have different names. For example, solving 65 + 59 as 50 + 50 + 5 + 9 may be using Make Hundreds thinking, Compensation thinking, or Partial Sums thinking. The focus must be on the ideas (not the naming of the strategy).

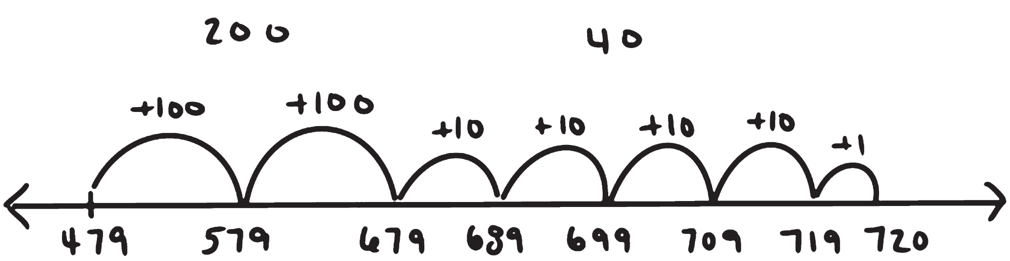
FLUENCY ACTION 2: Solve in a Reasonable Amount of Time

There is no set amount of time that should be expected for solving any whole number addition or subtraction problem. Students should be able to work through a problem without getting stuck or lost. The amount of time is relative to the student's grade and mathematical maturity. Keep in mind that appropriate strategies can be carried out in inefficient, unreasonable ways. For example, Figure 3a shows counting on in chunks. But a student might count on by doing 24 jumps of tens and ones, which is not a method to solve 479 + 241 in a reasonable amount of time.

FLUENCY ACTION 3: Trade Out or Adapt a Strategy

As strategies are better understood, students are able to adapt them or swap them out for another more efficient strategy. Students first learning to use the Count On strategy may think of 241 as 24 jumps of 10. They should soon begin to *adapt* that strategy by counting in larger chunks as shown in Figure 4.

FIGURE 4 ● Counting by Hundreds, Tens, and Ones



Rather than adapt the Count On strategy, a student might *trade out* the strategy for a Make Hundreds strategy, reasoning that they can move 21 over to rethink the sum as 500 + 220 (see Figure 3c). Notice that this Fluency Action is one of three connected to *reasonableness*. Within this action is noticing how the use of the strategy is going. If it isn't going well or if a student is getting bogged down, then the strategy needs to be adapted or traded for another, more efficient option.

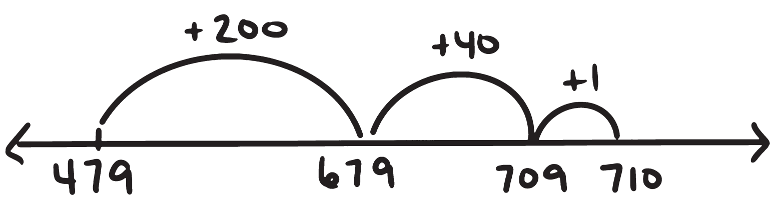
FLUENCY ACTION 4: Apply a Strategy to a New Problem Type

The Use Hundreds work shows how, when adding 479 + 241, 241 can be decomposed in a clever way to make 500 from 479. That individual's first experience with Make a 10 was likely to work with basic facts that use 10 (e.g., 9 + 7). They then learned to use the Make Tens strategy when adding 69 + 17 (69 + 1 + 16) and eventually to use the Make Hundreds strategy to make 500 when adding 490 + 27 (490 + 10 + 17). Their foundational understanding of Make a 10 was expanded and applied to new problem types. In time, this strategy will be applied to using a whole when adding fractions or decimals.

FLUENCY ACTIONS 5 AND 6: Complete Steps Accurately and Get Correct Answers

These two Fluency Actions are about accuracy. An error at the end of a problem may be due to an error in how a strategy was enacted. For example, in using Compensation for 64 – 29, a student changes the problem to 64 – 30 to get 34 and then *subtracts 1* from the answer (resulting in the incorrect answer of 33) rather than *adds 1* (resulting in the correct answer of 35). This incorrect answer is due to a misconception of the steps in implementing the Compensation strategy. Conversely, a student may enact a strategy accurately but make a computational error. The student's work in Figure 5 is a good example of this.

FIGURE 5 ● Counting On Incorrectly



Fluency Action 6 is one of three connected to *reasonableness*. Within this action is noticing if your answer makes sense. While reasonableness has been woven into the discussion of Fluency Actions, it is critical to fluency and warrants more discussion.

REASONABLENESS

As described earlier, reasonableness is more than "checking your answer." Reasonableness occurs in three of the six Fluency Actions as shown in Figure 2 and described within the related Fluency Actions. Let's revisit the sum 479 + 241. Counting on to solve 479 + 241 is a reasonable strategy choice (Action 1). Carrying it out reasonably means to add on chunks of 241 (a student should notice that if they are counting by tens, the strategy is not going well and adapt their approach) (Action 3). Finally, 720 is a reasonable answer (Action 6). Checking to see if an answer is reasonable can be determined in (at least) three different ways:

1. You might estimate the sum to be about 700 by rounding each addend.
2. You might estimate it to be about 740 using friendly numbers (500 and 240).
3. You might think about it being in the range of 600 (400 + 200) and 800 (500 + 300).

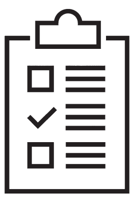
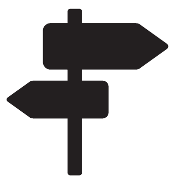
It takes time to develop reasonableness. It should be practiced and discussed as often as possible. Students can develop reasonableness by practicing three moves (a match to the Fluency Actions 1, 3, and 6).

THREE 'Cs' OF REASONABLENESS

Choose: Choose a strategy that is efficient based on the numbers in the problem.
Change: Change the strategy if it is proving to be overly complex or unsuccessful.
Check: Check to make sure the result makes sense.

You can encourage and support student thinking about reasonableness by providing Choose, Change, Check reflection cards (see Figure 6). These cards can be adapted into anchor charts for students to use while working on problems or during class discussions about adding and subtracting.

FIGURE 6 ● Choose, Change, Check Reflection Card for Students

CHECKS FOR REASONABLENESS		
Choose	Change	Check
		
Is this something I can do in my head? What strategy makes sense for these numbers?	Is my strategy going well or should I try a different approach? Does my answer so far seem reasonable?	Is my answer close to what I anticipated it might be? How might I check my answer?

Icon sources: Choose by iStock.com/Enis Aksoy; Change by iStock.com/Sigit Mulyo Utomo; Check by iStock.com/Indigo Diamond.

online resources This resource can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

WHAT FOUNDATIONS DO STUDENTS NEED TO DEVELOP FLUENCY WITH ADDITION AND SUBTRACTION?

To develop fluency, students need a strong foundation in five domains:

- *Conceptual understanding*: knowing the meaning of the operations
- *Properties*: being able to use the operations in order to manipulate numbers and retain equivalence
- *Utilities*: small skills that make a big difference, such as knowing how far 9 is from 10
- *Computational estimation*: being able to quickly and flexibly determine a "close" answer
- *Basic facts*: single-digit addition and subtraction facts that are needed for multidigit work

Rushing students to strategy instruction before these foundations are firmly in place can be disastrous.

CONCEPTUAL UNDERSTANDING

TEACHING TAKEAWAY

Developing fluency begins with stories and contexts. It is a mistake to save story problems as an application, as stories give students a context from which they can reason.

Developing fluency from conceptual understanding begins with including concrete experiences for students so they can make sense of the quantities. Hence, developing fluency *begins* with stories. It is a mistake to save story problems as an application, as stories give students a context from which they can reason.

Story problems must vary in two ways:

1. Type of situation (a join story, a part–part–whole story)
2. What is unknown in the story (the initial quantity, the change, or the result)

If you audit a page of "word problems" for addition and subtraction, you will likely find that "join" stories for addition and "separate" stories for subtraction are over-represented, and part–part–whole and compare stories are under-represented. Audit again for what is unknown in the story and you will likely find that most or all of the stories read as "result unknown." When stories fall into the same style, students overgeneralize how to interpret the stories. Figure 7 can be used as a resource to be sure you are varying your story types (for example, you can tally which types of stories you are telling as part of action research).

FIGURE 7 ● Addition and Subtraction Situations

ADDITION AND SUBTRACTION SITUATIONS			
Join	**Separate**	**Compare**	**Part–part–whole**
Story is about something being added to an original amount.	Story is about something being removed from the original amount.	Story compares two quantities.	Story is about combining different types of objects.
Ex: AJ had ____ books. She got ____ books. She has a total of ____ books.	Ex: AJ had ____ books. She gave away ____ books. Now she has a total of ____ books.	Ex: AJ has ____ books. Ian has ____ books. AJ has ____ more/less than Ian.	Ex: AJ has ____ fiction books and ____ nonfiction books. The total of her books is ____.

Beyond considering story type and missing values, here are few important ideas to keep in mind:

1. Stories need to be *relevant* to students, meaning that students are familiar with the context and it is interesting to students.
2. Avoid key-word strategies for solving story problems so that students must make sense of the problem and how addition or subtraction can be used to find a solution.
3. Use representations that are true to the situation (e.g., if comparing heights, use a vertical number line) until students have a deep understanding and are able to work symbolically and move between representations and tell explicitly how they are related.

In addition to stories, visuals provide concrete, conceptual beginnings for students. For example, young children count collections of objects in order to start learning to skip-count (Franke, Kazemi, & Turrou, 2018). Counting objects progresses to counting visuals and representations such as a Hundred Chart or number line, which eventually leads to abstract counting strategies such as the Count On/Count Back strategy. Hundred Charts and number lines similarly support the other reasoning strategies, such as Make Tens (or Hundreds) and Compensation.

PROPERTIES AND UTILITIES FOR STRATEGIC COMPETENCE

In addition to conceptual foundations, fluency is grounded in using properties of the operations and a few other skills that we refer to as "utilities" because students must utilize them in their reasoning. First, fluency with addition relies heavily on students using the commutative and associative properties of addition. Note that knowing properties does not equal using properties. It is *not* useful to have students simply name the associative property. It is absolutely necessary that students *utilize* this property in solving problems efficiently.

TEACHING TAKEAWAY

Knowing properties does not equal using properties.

For example, in the following problems, students look to rearrange these numbers (mentally or in writing) as they look for pairs that add to benchmark numbers:

$$76 + 38 + 24 \qquad\qquad 542 + 702 + 258 + 498$$

$$100 + 38 = 138 \qquad\qquad 800 + 1{,}200 = 2{,}000$$

The first problem is the commutative property in use:

$$76 + 38 + 24 = 76 + 24 + 38$$

The second problem uses both the commutative and associative properties:

$$\begin{aligned} 542 + 702 + 258 + 498 &= 542 + 258 + 702 + 498 \\ &= (542 + 258) + (702 + 498) \\ &= 800 + 1{,}200 \end{aligned}$$

See Chapter 3 (pp. 47–75) of *Figuring Out Fluency* for more about foundations and good beginnings for fluency.

Beyond the properties is a short list of utilities that support fluency, which is presented in Figure 8.

FIGURE 8 ● Utilities for Strategic Competence With Addition and Subtraction

UTILITY	WHAT IT IS	RELATIONSHIP TO FLUENCY
Distance From a 10	Knowing that 9 is 1 away from 10 (and 8 is 2 away and so on).	Knowing how far a number (e.g., 5, 6, 7, 8, 9) is from 10 is necessary to implement reasoning strategies, in particular Counting On/Back/Up, Make Tens, and Use Compensation.
Composing and Decomposing Numbers Flexibly	Understanding diverse, flexible ways to compose and decompose, including but not limited to place value decomposition.	Flexibly decomposing numbers supports strategy selection and facility with any of the strategies.
Part–Part–Whole	Understanding that addition and subtraction are based on parts making up a whole.	While the label has two parts and one whole, the reality is that it could be three parts or more. In other words, a part can be broken into parts in order to make a 10 or 100, or other benchmark number that will make adding or subtracting easier.
Skip-Counting	Skip-counting by tens, hundreds, and thousands, including multiples of these.	Efficiency comes from skip-counting in chunks rather than singles. This applies to different strategies, including Counting On/Back/Up and Partial Sums.

COMPUTATIONAL ESTIMATION

Just like computation, there are strategies for estimation and the use of those strategies should be *flexible*. For addition and subtraction of whole numbers, students might use any of these methods:

1. *Rounding*: Flexible rounding means that one or both numbers might be rounded. Students may round to the nearest number or they may round one number up and one number down to have a more accurate estimate.

Rounding is a well-known strategy but is often approached in a step-by-step manner, which can interfere with the point of estimating—getting a quick idea of what the answer will be close to. Use conceptual language, such as, "Which tens/hundreds/thousands is that number close to?" Help students understand that they choose how to round. For example, rounding both 2,403 + 3,495 down to the nearest thousand will give a very low estimate, whereas rounding one up and one down gives a closer estimate.

2. *Front-end estimation*: In its most basic form, students just add or subtract the largest place value. More flexibly, though, students may use the largest two place values, or adjust their estimate because of what they notice with the rest of the numbers. Front-end estimation is *quick*. For example, 74 – 36 is 40 and 387 + 635 is 900. To adjust, take a quick look to the right and decide to keep the estimate or adjust. For 387 + 635, there's at least 100 more, so go up to 1,000.

3. *Compatible numbers*: With flexibility in mind, students change one or both of the numbers to a nearby number so that the numbers are easy to add or subtract. For example, estimating the sum of 283 and 725 could be 300 and 700 to make 1,000, or it could be 275 and 725, which is also 1,000. Compatibles are particularly useful with subtraction. Consider estimating 3,456 – 1,890. Compatible alternatives might be 3,400 – 1,400, 3,500 – 1,500, or 3,500 – 2,000.

DEVELOPING BASIC FACT FLUENCY

The teaching of basic facts must attend to conceptual understanding and strategies for reasoning rather than rote instruction (Bay-Williams & Kling, 2019; O'Connell & SanGiovanni, 2015). Reasons to focus on *strategies* when teaching the basic facts (as opposed to memorizing) include the following:

1. It is well established across many studies that students actually learn and retain their facts better when they focus on conceptual understanding versus memorization. In fact, students don't just learn and retain their facts better, they perform better in math *in general* (e.g., Baroody, Purpura, Eiland, Reid, & Paliwal, 2016; Brendefur, Strother, Thiede, & Appleton, 2015; Jordan, Kaplan, Ramineni & Locuniak, 2009; Locuniak & Jordan, 2008; Purpura, Baroody, Eiland, & Reid, 2016).
2. Students need to know and use these strategies to support whole number addition and subtraction (as well as decimal and fraction operations!). In our *Figuring Out Fluency* anchor book, we elaborate more on the key strategies and ideas for effectively developing basic fact fluency.
3. Students who learn to use and choose strategies for basic facts develop confidence. Students who memorize often develop anxiety. A student who knows how to generate an answer to a sum such as 8 + 7 (beyond counting) doesn't have to worry if they forget the fact. This sense of agency is critical to student success in mathematics!

Figure 9 lists the basic fact strategies for addition and subtraction. To be clear, automaticity is the goal for learning basic facts. Students become automatic through learning the strategies and practicing them over and over again. In so doing, students develop automaticity with the facts and with implementing the strategies. Examples are illustrated in Figure 10. Keep in mind that there are many ways to implement a reasoning strategy, and only one way is shown for each example.

FIGURE 9 ● Reasoning Strategies for Basic Fact Addition and Subtraction

STRATEGY NAME	HOW THE STRATEGY WORKS	EXAMPLE STUDENT TALK
Addition	**Example: 8 + 6**	
Near Doubles	Student looks for a double they know that is similar to the problem. In this case, 8 + 8, 6 + 6, or even 7 + 7.	That's 14: 6 plus 6 plus 2.
Making 10	Student moves some from one addend to the other so that one addend is 10.	It's 14. I moved 2 over and thought 10 + 4.
Pretend-a-10 (Compensation)	Student pretends the larger addend is 10, adds, then adjusts the answer.	It's 14. Well, 10 and 6 is 16, and I have to take two away, so that's 14.
Subtraction	**Example: 14 − 9**	
Think Addition	Student thinks how to get from the subtrahend (9) to the minuend (14) [9 + ____ = 14]. *Note: Subtraction as compare*	It's 5. I pictured a number line and jumped up 1 to 10 and then 4 more.
Down Under 10	Student jumps from the minuend (14) to 10 and then jumps the rest of the subtrahend (9). *Note: Subtraction as take away*	It's 5. I broke 9 into 4 and 5. I jumped down 4 to 10, and then 5 more to 5.
Take From 10	Student subtracts the subtrahend (9) from 10, then adds on the extra ones from the minuend. *Note: Subtraction as take away*	I got 5. I thought of 14 as 10 and 4, subtracted 9 from 10 and got 1, and added the 4 back on and it's 5.

Except for Near Doubles, each of these strategies evolves directly to the strategies that are the focus of Part 2 of this book when students deal with whole numbers. You can see this transformation in the examples in Figure 10.

FIGURE 10 ● How Basic Fact Strategies Grow Into General Reasoning Strategies

REASONING STRATEGY	EXAMPLES WITH MULTIDIGIT NUMBERS
Making 10 becomes **Make Tens** (Hundreds, etc.)	39 + 28 = 40 + 27 97 + 35 = 100 + 32 395 + 784 = 400 + 779
Pretend-a-10 becomes **Compensation**	39 + 28 → 40 + 30 − 3 → 70 − 3 → 67 3,499 + 5,148 → 3,500 + 5,148 − 1 → 8,648 − 1 → 8,647
Think Addition becomes **Counting Up (or Back)**, in general, finding the difference between the two numbers	89 − 75 → 75 to 85 (10) to 89 (4) → jumps add to 14 615 − 582 → 582 to 600 (18) to 615 (15) → jumps add to 33
Down Under 10 becomes **Counting Back**, in general using skip-counting to take away an amount	52 − 8 → 52 − 2 (to 50) − 6 → 44 52 − 28 → 52 − 2 (to 50) − 20 (to 30) − 6 (to 24) 3,450 − 1,650 → 3,450 − 450 (to 3000) − 1,200 (to 1,800)
Take From 10 becomes **Partial Differences**, with the subtrahend taken from the largest place value. This becomes flexible and blends in **Compensation**, too.	52 − 28 → 50 − 28 (22) → 2 + 22 (24) 456 − 280 → 400 − 280 → 120 + 56 → 176

WHAT AUTOMATICITIES DO STUDENTS NEED BEYOND THEIR BASIC FACTS?

Unlike the foundation of conceptual understanding, automaticities are not prerequisites for, but coincide with strategy instruction. For example, automaticity with basic facts (just discussed) begins with strategy instruction and leads to eventual automaticity with the facts. But there are automaticities beyond the basic facts that support student reasoning!

Automaticity is the ability to complete a task with little or no attention to process. Little thought, if any, is given to skills that are automatic (Cheind & Schneider, 2012). We consider automaticities to be those skills that a fluent person can do without much attention to process. For example, you know that if you need to break 2 off of a 6, there is 4 left. Subtraction or a number line is not needed; it is intuitive or reflexive. Figure 11 identifies automaticities that complements strategies for adding and subtracting whole numbers. Of course, this is not a complete list. These automaticities are strengthened through strategy instruction (and conversely, having these automaticities strengthens students' capacities to use strategies).

See Chapter 5 (pp. 107–129) of *Figuring Out Fluency* for more about automaticities for fluency.

FIGURE 11 ● Automaticities for Adding and Subtracting Whole Numbers

AUTOMATICITY	WHAT IT IS	HOW IT COMPLEMENTS STRATEGY INSTRUCTION?
Basic facts	Quickly recognizing how a problem relates to a basic fact (e.g., 3 + 8 relates to 30 + 80).	Identifying relationships to basic facts helps students consider which numbers to decompose and how to decompose them.
Breaking apart all numbers through 10	Being able to quickly decompose any number through 10, which includes breaking the number into more than two parts.	Decomposing numbers flexibly and with automaticity helps students select and carry out a strategy.
Base-10 combinations	Being able to quickly find combinations of 10 and multiples of 10, including hundreds.	Recognizing combinations of tens and multiples enables students to accurately carry out strategies (especially Make Tens/Hundreds, Use Partials, and Compensation).

WHAT ARE THE SIGNIFICANT STRATEGIES FOR ADDING AND SUBTRACTING WHOLE NUMBERS?

Teaching strategies beyond the common algorithms has been a challenge, as there has been pushback and criticism from families and in social media. Two questions require attention:

1. Why do students need strategies when they can use the standard algorithm?

One way to quickly respond to this question is to share an example for which the standard algorithm takes much more time than an alternative. For example, 98 + 99 or 301 – 295. Why learn other methods? Because many problems can be solved more efficiently another way. Fluent students look for efficient methods; if students are limited in the methods they are taught, they have little to choose from, which limits flexibility and efficiency.

2. What strategies are worthy of attention?

Let's just take some pressure off here. The list is short, and we must help students see that they are not necessarily learning a *new* strategy, but they are applying a strategy they learned with basic facts and transferring it to other numbers. In *Figuring Out Fluency* we propose Seven Significant Strategies. Of these, five relate to adding and/or subtracting whole numbers and they are listed in Figure 12.

TEACHING TAKEAWAY

Students don't need to constantly learn dozens of new strategies, but rather connect how the key strategies they learned for basic facts are transferred to other numbers.

FIGURE 12 ● Reasoning Strategies for Adding and Subtracting Whole Numbers

REASONING STRATEGIES	RELEVANT OPERATIONS
1. Count On/Count Back (Module 1)	Addition and Subtraction
2. Make Tens (Module 2)	Addition
3. Partial Sums and Differences (Module 3)	Addition and Subtraction
4. Compensation (Module 4)	Addition and Subtraction
5. Think Addition (Module 5)	Subtraction

To be clear, using connecting cubes or a number line is *not* a strategy. It is a representation. If a student subtracts using connecting cubes or a number line, they are implementing a strategy—perhaps Count Back or Think Addition. When a student says, "I used a number line," ask *how* they used it—then you will learn what strategy they used. Teaching for fluency means that each of these strategies is explicitly taught to students. We teach students to *use* the strategy, and then we give students many opportunities to engage in *choosing* strategies (Part 3 of this book). Explicitly teaching a strategy does not mean turning the strategy into an algorithm. Strategies require flexible thinking. Each module provides instructional ideas and practice to ensure students become adept at using each strategy flexibly. There is also a module on **standard algorithms** (Module 6), so that they are integrated into the use of strategies.

HOW DO I USE THE PART 2 MODULES TO TEACH, PRACTICE, AND ASSESS STRATEGIES?

Part 2 is a set of modules, each one focused on understanding why a specific reasoning strategy works and learning how to use it well. The six modules in Part 2 each have a consistent format. First, each module provides an overview of the strategy—unpacking what it is, how it works, and when it is useful. Then, each module provides a series of teaching activities for explicit strategy instruction, followed by a collection of practice activities, including routines, games, and centers.

EXPLICIT STRATEGY INSTRUCTION

Strategies must be explicitly taught so that students understand them and can use them. Each module provides teaching activities for explicit instruction. The activities are designed so that you can modify and extend them as needed. Any one activity might form the focus of your instruction over the course of multiple lessons. Keep in mind that you can swap out tools and representations as well as adjust the numbers within the task.

The last teaching activity in each module is a collection of *investigation prompts* that you can use to develop reasoning and understanding of the strategy. Each investigation prompt itself can easily become a core instructional task.

We intend for students to work with instructional activities in collaborative partner or group settings. We encourage you to let students make their own meaning and to make mistakes. After students engage in the activities, a group discussion is needed to focus student thinking on the concepts within the strategy, how the strategy works, and the different ways a strategy might be carried out.

QUALITY PRACTICE

Students need access to quality practice that is not a worksheet. Quality practice is focused on a strategy, varied in type of engagement, processed by the student to make sense of what they did, and connected to what they are learning.

Each practice section begins with **worked examples.** Worked examples are opportunities for students to attend to the thinking involved with a strategy, without solving the problem themselves. We feature three types to get at all components of fluency:

See Chapter 6 (pp. 130– 153) of *Figuring Out Fluency* for more about quality practice.

1. *Correctly worked example*: efficiency (selects an appropriate strategy) and flexibility (applies strategy to a new problem type)
2. *Partially worked example*: efficiency (selects an appropriate strategy) and accuracy (completes steps accurately; gets correct answer)
3. *Incorrectly worked example*: accuracy (completes steps accurately; gets correct answer)

Also, comparing two correctly worked examples is very effective in helping students learn to choose efficient methods. Throughout the modules are dozens of examples, which can be used as worked examples (and adapted to other similar worked examples). Your worked examples can be from a fictional "student" or authentic student work. Some of the prompts from teaching the strategy section are, in fact, worked examples.

The remaining practice activities include routines, games, and centers. Each activity provides a brief "About the Activity" statement to help you quickly match what your students need with a meaningful activity. General resources, including number cards, mini ten-frame cards, addition charts, and more, are also available for download on the companion website.

ASSESSING STRATEGY USE

Each module offers a plethora of practice activities. As students are practicing, you can observe and assess the extent to which they are able to apply the selected strategy. **Observation tools** help you keep track of where each student is and monitor their progress. An observation tool can be simple, such as a class list with an extra column. Your observations can be codes:

+ Is regularly implementing the strategy adeptly

✔ Understands the strategy, takes time to think it through

– Is not implementing the strategy accurately

A note-taking observation tool provides space for you to insert notes about how a student is doing (see Figure 13). You can laminate the tool and use dry-erase markers to reuse it for different observations, use sticky notes, or just write in the boxes.

FIGURE 13 ● Example Note-Taking Observation Tool

Strategy: Compensation to subtract   Type of Numbers: within 100

R. Abrams • Uses a number line • Is flexible • Needs time	R. Bailey Has difficulty choosing how to change numbers	G. Brooks + Explaining to partner a good way
H. Duncan Always changes subtrahend, then adjusts answer	M. Foote Used subtract too much and add back—ask her to share	M. Li
C. Miller Solving mentally with ease	W. Moore Always changes both numbers before subtracting	P. Muñoz
M. Roberts Has difficulty choosing how to change numbers	M. Rodriguez	S. Sanchez Always changes both numbers before subtracting

online resources This resource can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

Some days, you collect data on some students; other days, you collect data on other students. The data can help in classroom discussions and in planning for instructional next steps.

Journal prompts provide an opportunity for students to write about their thinking process. Each module provides a collection of prompts that you might use for journaling. You can modify those or easily craft your own. The prompt can specifically ask students to explain how they used the strategy:

> **Explain and show how you can use Make Hundreds to solve this problem: 637 + 693**

Or a prompt can focus on identifying when that strategy is a good idea:

> **Circle the problems that are good choices for solving with the Make Hundreds strategy and tell why you selected them:**
>
> 495 + 347   327 + 726   489 + 278   399 + 298

Interviewing is an excellent way to really understand student thinking. You can pick any problem that lends to the strategy you are working on and write it on a note card (or record two or three on separate notecards). While students are engaged in an instructional or practice activity, roam the room, select a child, show them a card, and ask them (1) to solve it and (2) explain how they thought about it. You can pair this with an observation tool to keep track of how each student is progressing.

HOW DO I USE PART 3 TO SUPPORT STUDENTS' FLUENCY?

As soon as students know more than one way, it is time to integrate routines, tasks, centers, and games that focus on choosing when to use a strategy. That is where Part 3 comes in. As you read in Fluency Action 1, students need to be able to choose efficient strategies. The strategy modules provide students *access* to those strategies, ensuring the strategies make sense and giving students ample opportunities to practice those strategies and become adept at using them. However, if you stop there, students are left on their own when it comes to choosing which strategy to use when. It is like having a set of knives in the kitchen but not knowing which ones to use for slicing cheese or bread, cutting meat, or chopping vegetables. Like with food, some items can be cut with various knives, but other food really needs a specific knife.

Do not wait until after all strategies are learned to focus on when to use a strategy—instead weave in Part 3 activities regularly. Each time a new strategy is learned, it is time to revisit activities that engage students in making choices from among the strategies in their repertoire. Students must learn what to look for in a problem to decide which strategy they will use to solve the problem *efficiently* based on the numbers in the problem. This is *flexibility* in action, and thus leads to fluency.

IN SUM, MAKING A DIFFERENCE

Part 1 has briefly described factors that are important in developing fluency, and these ideas are important as you implement activities from the modules. We sum up Part 1 with the following five key takeaways:

1. Be clear on what fluency means (three components and six actions). This includes communicating it to students and their parents.
2. Attend to readiness skills: conceptual understanding, properties, utilities, computational estimation, and, of course, basic fact fluency.
3. Through activities and discussion, help students connect on the features of a problem and how that relates to good strategy options.
4. Reinforce student reasoning and choice selection, rather than focus on speed and accuracy. Getting the strategies down initially takes more time but eventually will become more efficient.
5. Assess fluency, not just accuracy.

TEACHING TAKEAWAY

Teaching for fluency means teaching strategies as core instruction, routinely practicing them, and offering opportunities for students to choose among strategies.

Time invested in strategy work has big payoffs—confident and fluent students! That is why we have so many activities in this book. Teach the strategies as part of core instruction, *and* continue to practice throughout the year, looping back to strategies that students might be forgetting to use (with Part 2 activities) and offering ongoing opportunities to choose from among strategies (with Part 3 activities).

PART 2

STRATEGY MODULES

MODULE 1

Count On/Count Back Strategy

STRATEGY OVERVIEW: Count On/Count Back

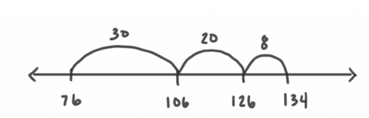
What is Count On? The Count On strategy begins with single digits (i.e., basic facts), counting on from one addend. For example, to add 8 + 5, start with 8 and count on 5 more. Count On extends this idea, using flexible skip counting to efficiently determine a sum. You start with either addend and add on the other number using convenient chunks.

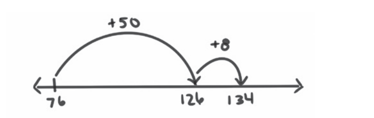
What is Count Back? The Count Back strategy also begins with single-digit problems, such as 9 – 4, where students start at 9 and count back 4. Count Back is used for subtraction in both "separate" (take away from the minuend) and "compare" (difference between the minuend and the subtrahend) problem situations. This module focuses on "separate" and Module 5 (Think Addition) focuses on "compare."

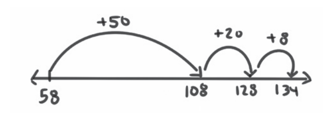
HOW DOES COUNT ON/COUNT BACK WORK?

Students choose benchmark numbers and keep track of partial results as they go. They may keep track in writing or mentally. In either case, they may use a series of equations or a number line to keep track. At first, students tend to use more steps (counts), but with experience, the chunks become more efficient. Flexibility is key.

COUNT ON OPTIONS USING AN OPEN NUMBER LINE



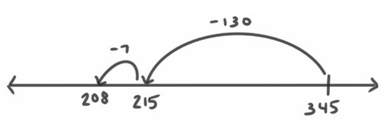


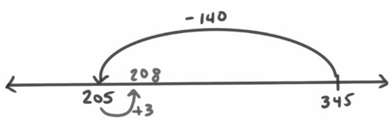


COUNT ON OPTIONS FOR 58 + 76 USING EQUATIONS

76 + 30 = 106 106 + 20 = 126 126 + 8 = 134	76 + 50 = 126 126 + 8 = 134	58 + 50 = 108 108 + 20 = 128 128 + 6 = 134

COUNT BACK OPTIONS USING AN OPEN NUMBER LINE





COUNT BACK OPTIONS FOR 345 – 137 USING EQUATIONS

345 – 100 = 245 245 – 30 = 215 215 – 7 = 208	345 – 130 = 215 215 – 7 = 208	345 – 140 = 205 205 + 3 = 208 [blends with Compensation]

WHEN DO YOU CHOOSE COUNT ON/COUNT BACK?

When one of the addends is relatively small, Count On is an efficient strategy (e.g., 4,579 + 215). Count Back is useful when the number being subtracted (subtrahend) is small. However, when a subtraction problem has a subtrahend close to the minuend (e.g., 436 – 385), a Think Addition or Count Up method is often more efficient.

COUNT ON AND COUNT BACK: Strategy Briefs for Families

It is important that families understand the strategies and know how they work so that they can be partners in the pursuit of fluency. These strategy briefs are a tool for doing that. You can include them in parent or school newsletters or share them at parent conferences. They are available for download so that you can adjust them as needed.

Count On Strategy

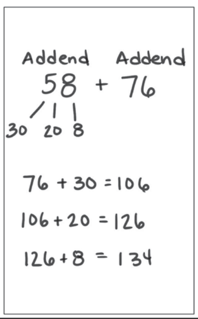
How It Works: To add, we can break apart an addend and count on from the other addend in chunks.

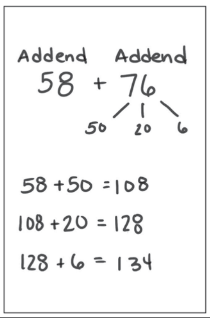
1. Choose which addend to break apart and which to count on from.
2. Break apart the addend into easy chunks.
3. Count on using the chunks you chose.

The left example shows that you can break apart 58 and count on from 76.

The right example shows that you can also break apart 76 and count on from 58.

When It's Useful: Count On is useful when you can easily break apart one of the addends into convenient chunks. These addends are typically small (two, three, or four digits).





Count Back Strategy

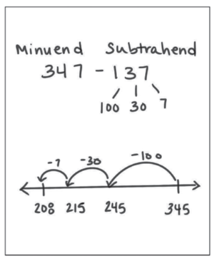
How It Works: To subtract, we can break apart the subtrahend (second number) and count back from the minuend (first number).

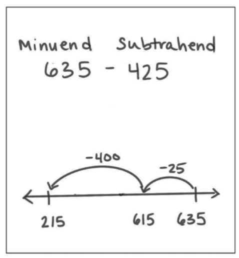
1. Break apart the subtrahend (the second number) into easy chunks.
2. Count back from the minuend (the first number) by the chunks.
3. The last count is the answer.

The left example shows how 137 can be broken into 100, 30, and 7. Note that those chunks can be counted in any order.

The right example shows how the person counts back 25 first and then 400.

When It's Useful: Count Back is useful when the number being subtracted is small. Count Back is also useful when the difference between the two numbers is small.





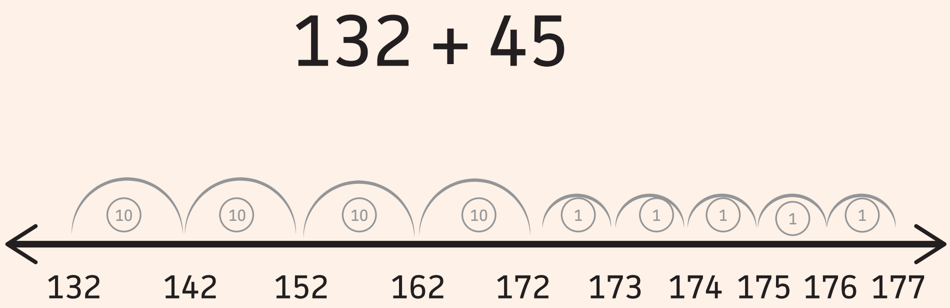
online resources These resources can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

TEACHING ACTIVITIES for Count On/ Count Back

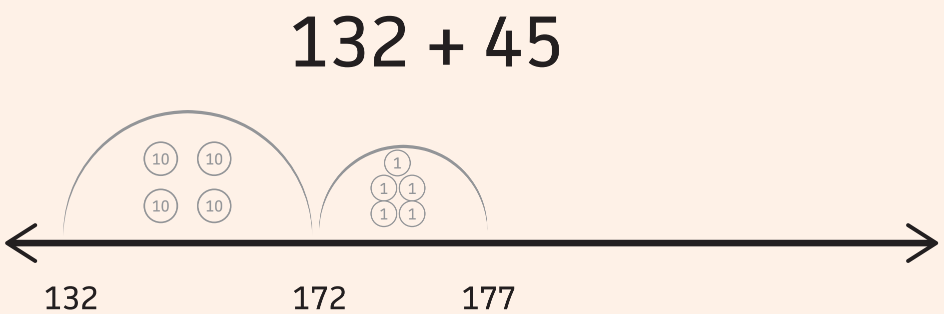
Count On and Count Back are often the first strategies students learn. What begins as count on or back by ones becomes count on by tens and then larger chunks like 30 or 100. In this section, you'll find instructional activities for helping students develop efficient ways to count on or count back. The goal is that students become adept at using counting strategies efficiently and accurately and also consider when they will want to use a counting strategy.

ACTIVITY 1.1 CONNECTING REPRESENTATIONS WHEN COUNTING ON

Count On is a strategy for finding the sum when addition is being used. One way to develop this understanding, and to model it, is to represent it with base-10 blocks, place value disks, and number lines. In this activity, have students show Count On with math tools such as place value disks or base-10 blocks. They record adding chunks on a number line or set of equations. The following image shows what this would look like for 132 + 45.



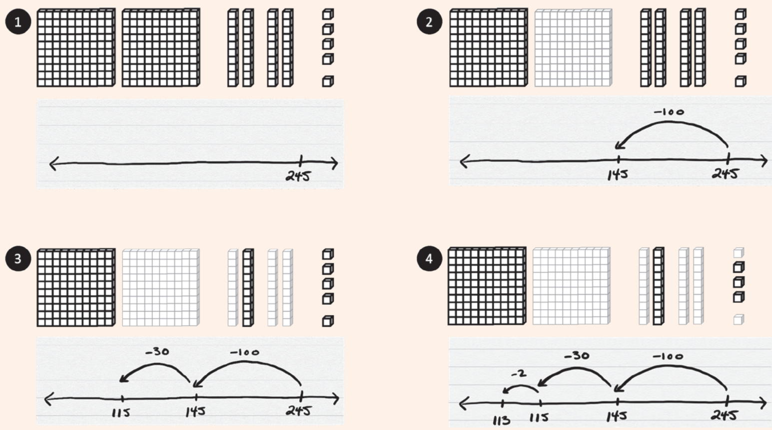
Students use place value disks to solve the problem.



Students later transition to chunking place value disks. They work toward using the least amount of chunks, making it a more efficient strategy.

ACTIVITY 1.2
CONNECTING REPRESENTATIONS WHEN COUNTING BACK

Count Back is a strategy for finding the difference when subtraction is being used to take away. A good way to develop this understanding, and to model it, is to represent it with base-10 blocks and number lines. In this activity, have students show Count Back with base-10 blocks and record taking away chunks on a number line or set of equations. The following image shows what this would look like for 245 – 132.



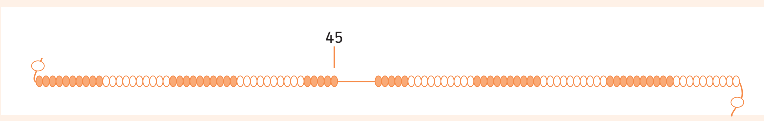
Be sure that students record each jump as they take away a certain amount. You also want students to record the equation that goes with the work. For step 2, the student would write 245 – 100 = 145. For step 3, the recording would be 145 – 30 = 115. The last recording would be 115 – 3 = 112. After students complete the problem, facilitate a discussion about the differences in how students counted back.

ACTIVITY 1.3
BEADED NUMBER LINES

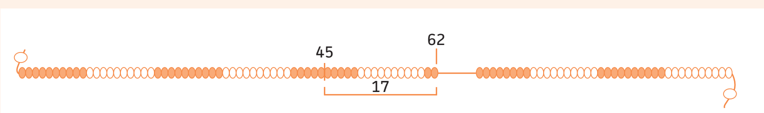
One way to develop the understanding of Count On and Count Back is to represent it with a 100 beaded number line and connect the work to a written number line. In this activity, students partition the beads to represent the first addend. Then students would shift beads to count on the amount of the second addend. This creates the sum. Students then record their work on a number line. Consider 45 + 17. Students move 45 beads to the left.

TEACHING TAKEAWAY

It can be helpful for students to use clothespins to keep the amount they're working with separate from the unused beads.



Then, students count on 17 more before counting a total.



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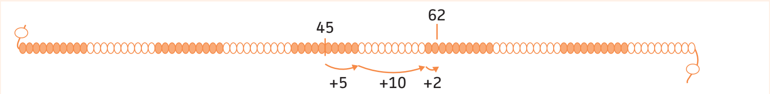
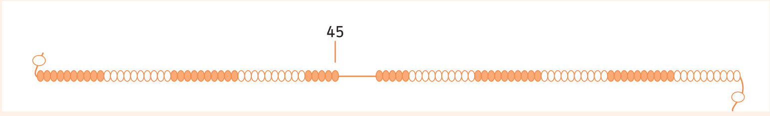
(*Continued*)

After using the bead counters, students record their thinking on a bead counter recording sheet.

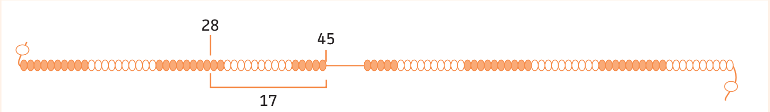


To count back, students slide 45 beads to the left to represent the minuend. A clothespin is handy here for keeping the minuend separate from the rest of the beads. The following example shows how this would unfold with 45 – 17.

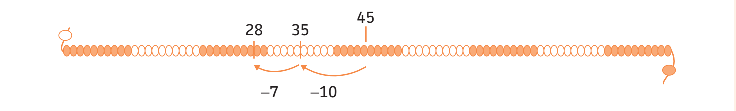
Students separate 45 from the entire set of beads.



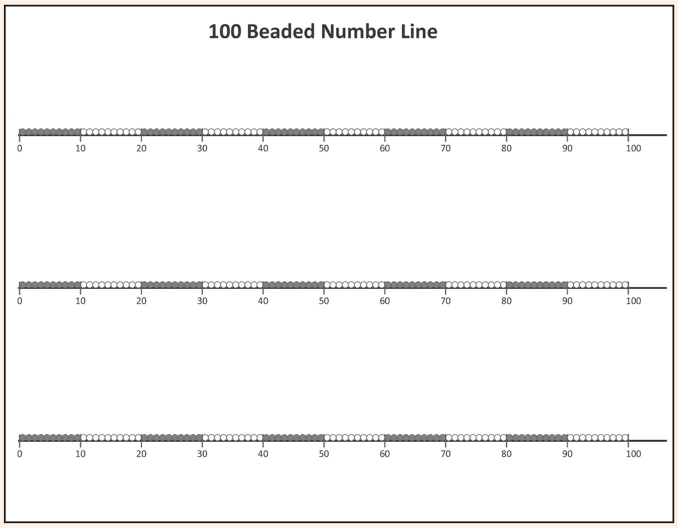
Then, students shift the beads (Count Back) that represent the subtrahend. This will create the final difference. Students then record their work on a number line.



After completing the work, students record their thinking on a bead counter number line.



RESOURCE(S) FOR THIS ACTIVITY

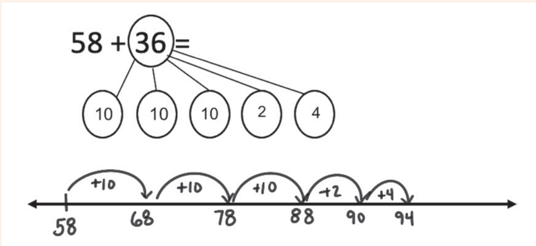


online resources This resource can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

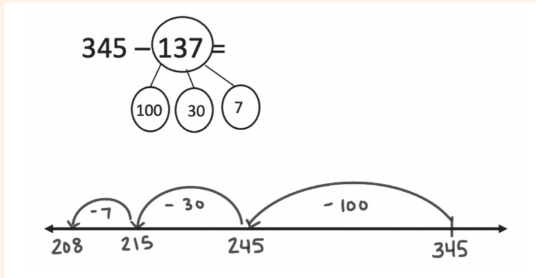
ACTIVITY 1.4
NUMBER BONDS FOR COUNT ON/COUNT BACK IN CHUNKS

Number bonds let students decompose numbers in useful ways. Students use the number bond to help them decompose a number into convenient parts. In the Count On version, after a problem is posed, students first determine which addend to count on from and which addend to break apart. Then students count on using the number line and the decomposed parts. In Count On example 58 + 36, a student thinks to break apart 36 into 3 tens, 5 ones, and another one and then shows those jumps on the number line. This activity also works well to develop the Count Back strategy. Students decompose the subtrahend and then count back from the minuend.

COUNT ON EXAMPLE



COUNT BACK EXAMPLE



As students decompose numbers, look for them to break a number in each 10 or chunks of 10. In Count Back example 345 – 137, the student makes a jump of 30 instead of three jumps of 10. You want to help students move from individual hundreds, tens, or ones to groups of each.

ACTIVITY 1.5
PROMPTS FOR TEACHING COUNT ON/ COUNT BACK

Use the following prompts as opportunities to develop understanding of and reasoning with the strategy. Have students use representations and tools to justify their thinking including base-10 models, number lines, number charts, and so on. After students work with the prompt(s), bring the class together to exchange ideas. These could be useful for collecting evidence of student understanding. Any prompt can be easily modified to feature different numbers (e.g., three-digit or four-digit numbers) and any prompt can be offered more than once if modified.

- Show two different ways to add 35 + 76 using a number line. Do they have the same sum? Explain your thinking.
- Andie counts on to solve 496 + 237. What number would you suggest she start with? How should she then count on from the number you suggest? Show your thinking with a number line or equations.
- Show two different ways to subtract 376 – 135 using a number line. Do they have the same difference? Explain your answer.
- Siobhan counts back to solve 705 – 183. What jump would you suggest Siobhan start with? How should she then count back from the number you suggest? Show your thinking with a number line or equations.

(*Continued*)

(*Continued*)

- Create a problem that you would solve by counting back. Show how you can use the Count Back strategy to solve your problem.
- Create a problem that you would solve by counting on. Show how you can use the Count On strategy to solve your problem.
- Emma solved 516 + 475 by thinking 516 + 100 + 100 + 100 + 100 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1. What is another way she might have thought about the problem?
- How are Count On and Count Back similar and how are they different?
- Give an example of when the Count On strategy would not be the most efficient strategy to use.
- Give an example of when the Count Back strategy would not be the most efficient strategy to use.

NOTES

PRACTICE ACTIVITIES for Count On/Count Back

Fluency is realized through quality practice that is focused, varied, processed, and connected. The activities in this section focus students' attention on how this strategy works and when to use it. The activities are a collection of varied engagements. The discussion you facilitate after an activity or the reflection prompts you attach to it should help students think about what they did mathematically, how they reasoned about the activity, and when the math they did (namely the strategy) might be useful. Debriefing should also help students see how the practice activity connects to recent instruction or how the strategy connects to other strategies they know. Game boards, recording sheets, digit cards, and other required materials are available as online resources for you to download, possibly modify, and use. As students work with activities, you want to look for how well they are acquiring the strategy and assimilating it into their collection of strategies.

FLUENCY COMPONENT	WHAT TO LOOK FOR AS STUDENTS PRACTICE THIS STRATEGY
Efficiency	• Are students using the Count On/Count Back strategy efficiently? (e.g., Are they counting efficiently?) • Do they use the strategy regardless of its appropriateness for the problem at hand? (e.g., Do they count by ones when one addend is a multiple of 10, like 40?) • Do they change their approach to or from this strategy as they begin to work the problem and realize the initial approach will be less efficient?
Flexibility	• Are students using the Count On/Count Back strategy in groups or singles? • Are students counting in appropriate ways or do they rely on the same (usually lower-level) counting approach? • Are students carrying out the strategy in flexible ways? (e.g., Do they sensibly choose what number to count on from?) • Do they change their counting approach if it doesn't seem to be working?
Accuracy	• Are students using the Count On/Count Back strategy accurately? • Are students accurate with certain counts (e.g., counting by groups of 10, counting around a decade or century)? • Are students finding accurate solutions? • Are they considering the reasonableness of their solutions?* • How is estimation worked into the practice?*

*This consideration is not unique to this strategy and should be practiced throughout the pursuit of fluency with whole numbers.

WORKED EXAMPLES

Worked examples are problems that have been solved. Correctly worked examples can help students make sense of a strategy and incorrectly worked examples attend to common errors.

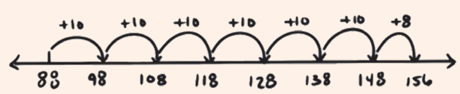
Related to the Count On/Count Back strategy, worked examples can help students see different options for skip counting and help them work toward efficient ways to count on or back. Incorrectly worked examples are based on common challenges or errors when using counting strategies, as seen in the following examples:

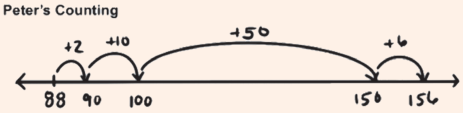
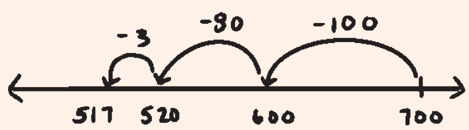
1. The student loses track of how many jumps and makes too many or too few jumps.
    - 871 – 355: counts back 771, 661, 551, 441, then continues jumps for tens and ones.
    - 394 + 477: starts at 400 and counts on to 877 but forgets to count back 6.

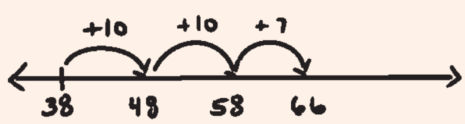
2. The student makes an error in breaking apart a number.
    - 87 + 71: from 87, counts on 20 to get to 107 and then 40 [rather than 50] to get to 147 and one more to 148.
    - 4,225 – 1,270: counts back 1,200 to 3,025, then back 25 to 3,000, then back 55 [rather than 45] to 2,945.
3. The student misses a count when consecutive digits are the same.
    - 871 – 449: counts back 4 hundreds to 471 but overlooks 4 tens because a jump of 4 was just made.

Although not an error, students may continue to count by single ones, tens, or hundreds instead of counting in chunks or multiples of these. Show a worked example that does this and ask students to consider how they might combine jumps to add/subtract more efficiently. Various worked examples can be found throughout this module, or you can collect authentic worked examples (see prompts from Activity 1.5, for example). Additionally, have students compare worked examples to highlight different ways to count on or back. A sampling of ideas are provided in the following table.

SAMPLE WORKED EXAMPLES FOR COUNT ON/ COUNT BACK

<table>
<tr>
<td>Correctly Worked Example
(make sense of the strategy)
What did ______ do?
Why does it work?
Is this a good method for this problem?</td>
<td colspan="2">Compare Michelle's and Peter's methods for adding 68 + 88:
Michelle's Counting


Peter's Counting

</td>
</tr>
<tr>
<td>Partially Worked Example
(implement the strategy accurately)
Why did ______ start the problem this way?
What does ______ need to do to finish the problem?</td>
<td>Nate's start for 546 – 175:
546 - 200 = 346</td>
<td>Jeannette's start for 546 – 175:
546 - 140 = 406</td>
</tr>
<tr>
<td>Incorrectly Worked Example
(highlight common errors)
What did ______ do?
What mistake does ______ make?
How can this mistake be fixed?</td>
<td colspan="2">Luke's work for 705 – 183:


Allie's work for 38 + 27:

</td>
</tr>
</table>

ACTIVITY 1.6

Name: "Or You Could" **Type:** Routine

About the Routine: Count On is most efficient when done in chunks. Students may be comfortable counting on by ones, tens, or hundreds instead of groups. This routine helps students work to use chunks by renaming expressions that add singles as expressions that add groups. In it, students are shown a basic (inefficient) approach to count on or count back. It asks them to think of another way they could carry out the strategy. Keep in mind that it is reasonable for students to first learn and use Count On and Count Back by decomposing a number into individual tens and ones and counting by each. The strategy is most efficiently used by counting on or back by chunks. This routine helps students develop more efficient approaches for counting.

Materials: list of two or three completed examples of Count On or Count Back

Directions:

1. Provide completed Count On or Count Back problems such as the following examples.
   - You can solve 26 + 45 by thinking 26 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 or you could …
   - You can solve 92 – 64 by thinking 92 – 10 – 10 – 10 – 10 – 10 – 10 – 1 – 1 – 1 – 1 or you could …
   - You can solve 516 + 145 by thinking 516 + 100 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 or you could …
   - You could solve 421 – 233 by thinking 400 – 100 – 100 – 10 – 10 – 10 – 1 – 1 – 1 or you could …
2. Ask students to talk with a partner about another way to count on or back.
3. After a few moments, bring the group together to share their thinking.
4. As students share more efficient ways to chunk the skip counts, record their thinking on a number line or with an equation.
5. Reinforce to students how the different approaches yield the same sum or difference.

To note, it's important to avoid using too many approaches. You want students to find and explain efficient approaches. In time, you can begin to modify the routine even further by providing a slightly more efficient chunking as shown in these two examples. Even though the hundreds were chunked, there is still an opportunity to chunk the tens and ones.

- You could solve 716 + 244 by thinking 716 + 200 + 10 + 10 + 10 + 10 + 4 or you could …
- You could solve 91 – 33 by thinking 91 – 1 – 10 – 10 – 10 – 2 or you could …

Keep in mind that sometimes chunking may be manipulated for friendlier computations. For example, in 378 + 344, one might add on 300 (678), then 2 (680), then 20 (700) to make a 10 and then a 100 and last add the remaining 22 (722). No matter how students think about chunking addends or subtrahends for Count On or Count Back, be sure to ask them to explain why that approach is efficient. Also, be sure that you accept their thinking and share other ideas but be careful to avoid saying that one way is "correct."

TEACHING TAKEAWAY

Ask students to explain why an approach is efficient but avoid saying one way is correct.

ACTIVITY 1.7

Name: "The Count" **Type:** Routine

About the Routine: This routine has students estimate and skip count. It is a good opportunity for practicing skip counting by a variety of intervals, which is essential for using the Count On and Count Back strategies.

Materials: Identify a counting interval and a starting number, provide a supporting number chart.

Directions:

1. Set a clear counting path so that students know how they will count in the room. Having students gather in a large circle is a good option.
2. Identify a starting number, the student who will count first, and a counting interval such as skip count forward by tens. Then, ask students about the impending count. Here are some questions to ask:
   - What are some numbers we will say as we count?
   - What number do you think you will say?
   - What will be the last number said?
3. As students count, you can record the numbers they say on the board or mark them on a related number chart or number line. Doing either will help a postcount discussion about the patterns within the numbers that were said and discussion about the predictions made before the count.
4. After the count, discuss patterns within the numbers said, challenges with counting, and how student predictions compared with the results of the count.

THE COUNT			
Start with 108. Count on by 20.	Start with 336. Count on by 50.	Start with 517. Count back by 10.	Start with 784. Count back by 5.

The example shows a variety of prompts you can pose. Note that counting backward is also an important use of the routine because it aligns with subtraction. Also note that there are a variety of intervals provided. Students should have experience working with a variety of skip counts to help them move from counting by singles to groups.

ACTIVITY 1.8

Name: "Strategize First Jumps" **Type:** Routine

About the Routine: This routine helps students consider different ways to skip count in order to efficiently solve a problem. It helps students think strategically about decomposing problems.

Materials: list of three to four problems on the same topic but that lend to different ways to count on/count back

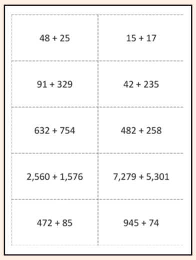
Directions: This routine involves showing a series of problems, one at a time, like a Number Talk (Parrish, 2014).

1. Ask students to mentally determine their first jump (only), and signal when ready (since it is only the first jump, they only need about 10–15 seconds).
2. Ask different students to share what they would do as their first jump. Record all ideas using a number line or Hundred Chart.
3. Ask students to pick one of the first jump ideas and finish the problem.
4. Share answers and discuss whether the first jump they chose worked out well or not.
5. Repeat with other problems (as time allows).
6. Conclude the series with a synthesizing discussion, asking questions such as these: What are efficient ways to count on (count back)? When will you use a Count On or Count Back strategy? When might counting not work very well?

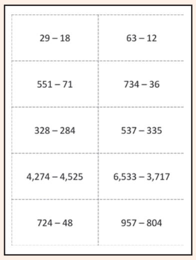
This routine can also focus *only* on first jumps (without finishing the problems). Then, more examples can be used in less time. Possible problems to use include the following:

48 + 25	91 + 329	632 + 754	2,560 + 1,576
15 + 17	42 + 235	482 + 258	7,279 + 5,301
29 – 18	734 – 36	328 – 284	4,274 – 4,525
63 – 12	551 – 71	537 – 335	6,533 – 3,717

RESOURCE(S) FOR THIS ACTIVITY



48 + 25	15 + 17
91 + 329	42 + 235
632 + 754	482 + 258
2,560 + 1,576	7,279 + 5,301
472 + 85	945 + 74



29 – 18	63 – 12
551 – 71	734 – 36
328 – 284	537 – 335
4,274 – 4,525	6,533 – 3,717
724 – 48	957 – 804

online resources These game cards can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.9

Name: Count On Bingo **Type:** Game

About the Game: *Count On Bingo* helps students think about the jumps they make and how they might become more strategic with their jumps. Blank 5 × 5 bingo boards can be downloaded and students can then create their own boards by writing the numbers 1–9 and multiples of 10 randomly anywhere on their board.

Materials: *Count On Bingo* boards (one per student), counters, expression cards (or a list of expressions)

Directions:

1. Players take turns pulling expression cards.
2. Players solve the expression on the card using the Count On strategy.
3. Then players place a counter on the jumps/chunks they used to solve their expression.
4. Bingo is five counters in a row in any direction.

For example, students pull the expression card 48 + 23. Player 1's jumps are 10 + 10 + 3 (on the left board) and they put two counters on the 10 and one counter on the 3. Player 2's jumps are 20 + 3 (on the right board) and they place a counter on those numbers. Note that player 1 stacked the counters on the 10 for this problem because they made two jumps of 10.

Count On Bingo

Directions: Choose from the numbers in the box. Write a number in each box. You can write a number more than once.

0 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90

10	4	30	20	10
6	10	40	10	3
70	2	90	5	30
6	20	7	70	80
10	10	1	8	40

Count On Bingo

Directions: Choose from the numbers in the box. Write a number in each box. You can write a number more than once.

0 1 2 3 4 5 6 7 8 9 10 20 30 40 50 60 70 80 90

40	3	20	10	50
8	10	3	1	20
7	10	20	30	40
6	5	70	6	8
1	10	10	90	4

You can find a list of expressions at the companion website that might be used for playing the game, although it will work with any examples. Expressions can be written on index cards if you want small groups of students to play *Count On Bingo* without you.

online resources Game cards and this game board can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.10

Name: Pick Your Jumps **Type:** Game

About the Game: *Pick Your Jumps* helps students choose the most efficient amount of jumps. Players win each round by solving a problem accurately with the fewest jumps.

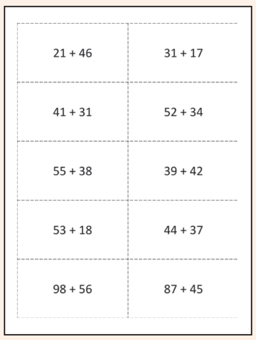
Materials: a short list of expressions or expressions recorded on index cards, *Pick Your Jumps* game board

Directions:

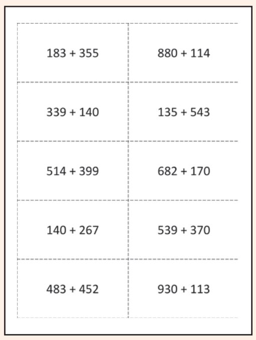
1. Players take turns pulling expression cards and solving the problem with a Count On or Count Back strategy.
2. Players solve the problem and record the number of jumps they made.
3. The player with the fewest number of jumps gets a point for the round. Note that the solution must be correct to earn a point.
4. The player with the most points at the end of four rounds wins.

For example, players turn over 21 + 46. Player 1 solves it by counting on from 21 by 10s (4 jumps) and jumping 6 (1 jump) to reach 67. Player 2 solves it by counting on from 46 by 10s (2 jumps) and jumping 6 (1 jump) to equal 67. Player 2 scores a point.

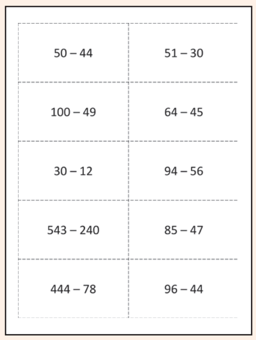
RESOURCE(S) FOR THIS ACTIVITY



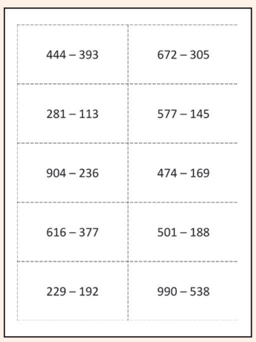
21 + 46	31 + 17
41 + 31	52 + 34
55 + 38	39 + 42
53 + 18	44 + 37
98 + 56	87 + 45



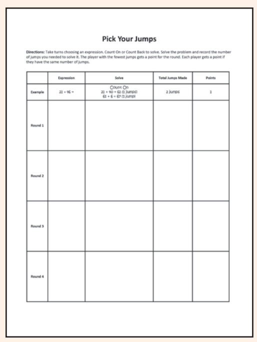
183 + 355	880 + 114
339 + 140	135 + 543
514 + 399	682 + 170
140 + 267	539 + 370
483 + 452	930 + 113



50 – 44	51 – 30
100 – 49	64 – 45
30 – 12	94 – 56
543 – 240	85 – 47
444 – 78	96 – 44



444 – 393	672 – 305
281 – 113	577 – 145
904 – 236	474 – 169
616 – 377	501 – 188
229 – 192	990 – 538



Pick Your Jumps

online resources Game cards and this game board can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.11

Name: Make It Close **Type:** Game

About the Game: *Make It Close* is a target-based game for practicing addition and developing number sense. The goal is to create a sum as close to the target as possible. The unique twist in this game is that the target changes from round to round.

Materials: four decks of digit cards (0–9) or playing cards (with aces equal to 1; face cards and 10s removed), *Make It Close* recording sheet

Directions:

1. Players deal four cards to make 2 two-digit addends. Both players use the Count On strategy to find the sum, which becomes the target for the round. In the example, the target for round 1 is 103.
2. Each player deals themselves four digit cards to make an addition problem with 2 two-digit addends.
3. The players arrange the four digits so that their sum is close to the target. For example, player 1 arranges digits for a sum of 109. Player 2 arranges digits for a product of 101.

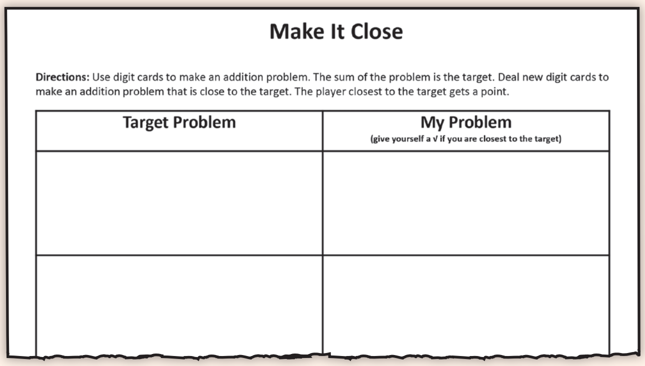
Target	4	2	+	6	1	103
Player 1	3	8	+	7	1	109
Player 2	4	9	+	5	2	101

4. The player with the sum closest to the target for round 1 gets a point. This is player 2 in the example.
5. The first player to earn 5 points wins the game.

TEACHING TAKEAWAY

Have students say aloud to their partner how they counted on to add.

RESOURCE(S) FOR THIS ACTIVITY


Make It Close

Directions: Use digit cards to make an addition problem. The sum of the problem is the target. Deal new digit cards to make an addition problem that is close to the target. The player closest to the target gets a point.

Target Problem	My Problem (give yourself a √ if you are closest to the target)

online resources This resource can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.12

Name: Find the Equation **Type:** Center

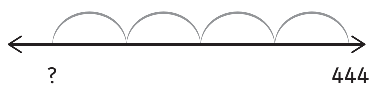
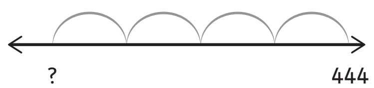
About the Center: This center helps students think about how they can count on or back. It also connects representations of number lines and equations. While described as a center, Find the Equation can also be used as an instructional activity.

Materials: open number line cards, Find the Equation recording sheet (one per student)

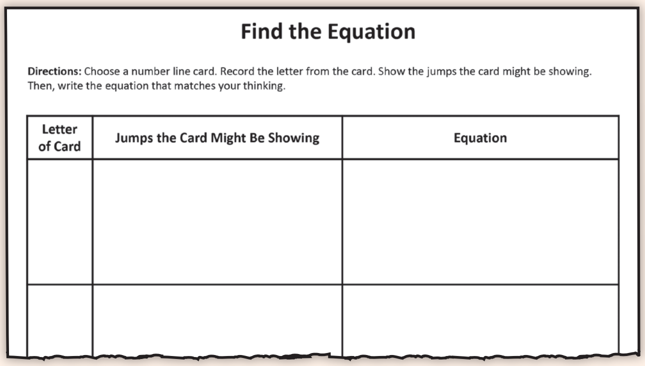
Directions:

1. Students select a number line card.
2. Students generate values for each jump and record the equation that the number line card might model.
3. Students repeat step 2 to show a different problem with the same number line card to show another problem it might model.

For example, a student picks the number line card shown.

	Student thinks of each jump as a jump of 10 and records: 444 – 10 = 434 434 – 10 = 424 424 – 10 = 414 414 – 10 = 404	Student records the equation: 444 – 40 = 404
	Student thinks of each jump as 2 jumps of 100 and 2 tens: 444 – 100 = 244 344 – 100 = 244 244 – 10 = 234 234 – 10 = 224	Student records the equation: 444 – 220 = 224

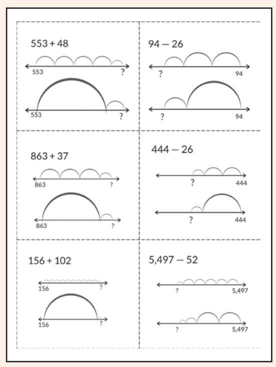
RESOURCE(S) FOR THIS ACTIVITY



Find the Equation

Directions: Choose a number line card. Record the letter from the card. Show the jumps the card might be showing. Then, write the equation that matches your thinking.

Letter of Card	Jumps the Card Might Be Showing	Equation



online resources Number line cards and this recording sheet can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.13

Name: Same But Different **Type:** Center

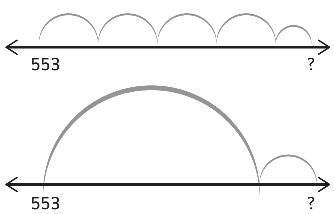
About the Center: This center helps students recognize that numbers can be added in different ways and to develop efficiency when counting on or counting back by connecting single counts to group counts.

Materials: open number line cards, Same But Different recording sheet

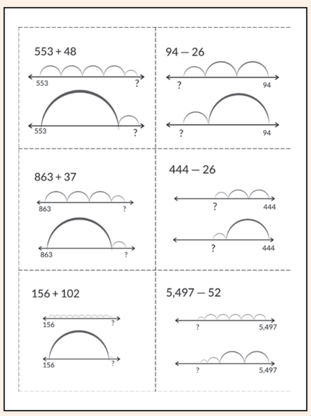
Directions:

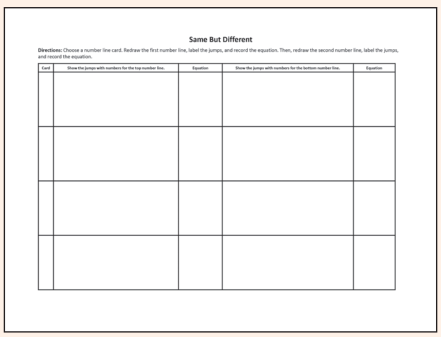
1. Students select an open number line card.
2. Students copy the image and record the value of jumps shown on the number line and the equation.
3. Students record the jumps on the related number line with jumps that represent chunks of ones, tens, or hundreds.
4. After working with the center, students should be asked to explain how the problems are the same and how they are different.

The following is an example.

553 + 48 	The student records 4 jumps of 10 and a jump of 8. The student records the equation 553 + 48 = 601.	The student records a jump of 40 and a jump of 8. The student records the equation 553 + 48 = 601.

RESOURCE(S) FOR THIS ACTIVITY





online resources Number line cards and this recording sheet can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.

ACTIVITY 1.14

Name: The Largest Sum, The Smallest Sum **Type:** Center

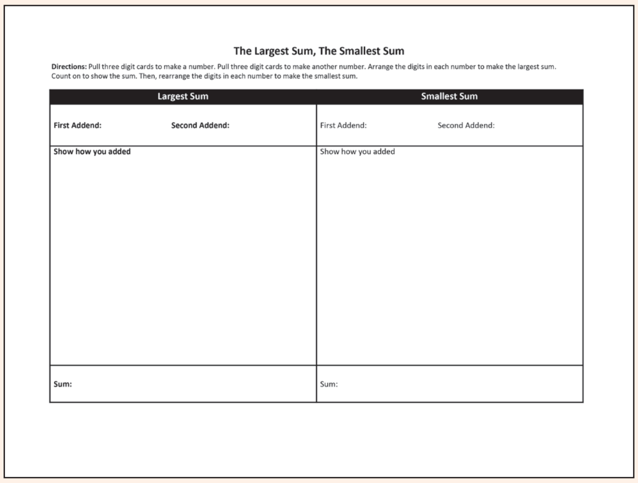
About the Center: This center reinforces Count On or Count Back strategies while also focusing on number sense and reasoning. It can be used as a game where students earn points for finding the largest or smallest sum within a group.

Materials: digit cards (0–9) or playing cards (queens = 0, aces = 1, kings and jacks removed); The Largest Sum, The Smallest Sum recording sheet

Directions:

1. Students pull three digit cards to make a three-digit number.
2. Students then pull three additional digit cards to make a second three-digit number.
3. Students arrange the digits in each addend to make the largest sum.
4. Students record the problem and use Count On to find the sum.
5. Students repeat the activity but this time, they rearrange the digits in each addend to make the smallest sum once again use Count On to find the sum.
6. Students record their thinking on the recording sheet or in their journals.

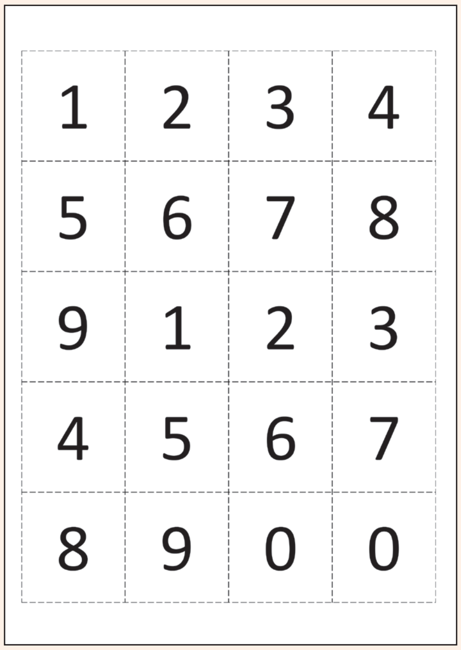
RESOURCE(S) FOR THIS ACTIVITY



The Largest Sum, The Smallest Sum

Directions: Pull three digit cards to make a number. Pull three digit cards to make another number. Arrange the digits in each number to make the largest sum. Count on to show the sum. Then, rearrange the digits in each number to make the smallest sum.

Largest Sum	Smallest Sum
First Addend: Second Addend:	First Addend: Second Addend:
Show how you added	Show how you added
Sum:	Sum:



online resources Digit cards and this recording sheet can be downloaded at **resources.corwin.com/FOF/addsubtractwholenumber**.