What Your Colleagues Are Saying . . .

If you have anything to do with transforming mathematics teaching and learning at your school, then stop everything right now and read this book. Karp, Dougherty, and Bush's *The Math Pact* strategically navigates the complex topic of creating and cultivating cohesive mathematics instruction by introducing readers to their Mathematics Whole School Agreement and specific elements that ensure school-wide success. Building on the authors' renowned *Rules That Expire* work, this book takes readers through the essential components of best practices of mathematics teaching. This should be a required read for any mathematics leadership program.

Hilary Kreisberg
Director
Center for Mathematics Achievement
Lesley University
Cambridge, Massachusetts

Just for a moment, imagine the positive and impenetrable mathematics energetic forcefield that could be created in a school if every teacher strategically and collaboratively decided upon agreements around mathematics vocabulary, notations, representations, and the overarching philosophy about how mathematics should be taught? Guess what! You do not have to imagine! No more reteaching, revising previously taught material, and explaining that "Well, in my classroom, we do it, say it, notate it this way." Because let's face it, those messages confuse students and families and can frustrate teachers. Grab this book, gather your colleagues, and get started in creating a unified and comprehensive whole school agreement that will positively enrich your students' mathematics learning experiences!

Beth Kobett
Professor
School of Education
Stevenson University
Board of Directors
National Council of the Teachers of Mathematics

As a fan of the "Rules That Expire" and "Whole School Agreement" articles, I consider this resource a welcome addition to my professional library! In this practical, easy-to-follow book, the authors provide educators with an extremely thorough and useful "go to guide" on establishing a Mathematics Whole School Agreement (MWSA). For those not familiar with the NCTM articles that initiated the movement, this resource defines what an MWSA is and why each school should establish one. More important, educators learn how to create and implement an MWSA, and the role each school and district stakeholder plays in implementation. This user-friendly resource provides teachers, teacher leaders, parents, and administrators with a comprehensive blueprint that includes research-informed practices, vignettes, grade-specific examples, and tools to encourage reflection.

Latrenda Knighten
Elementary Mathematics Instructional Coach
Baton Rouge, Louisiana

The Math Pact is a critical guide that takes stakeholders on a journey to create a Mathematics Whole School Agreement (MWSA). This journey begins with a look inward at mathematics instruction in their schools and then moves onward to compare their practices to best practices. Stakeholders arrive at their MWSA when they can ensure a unity of message that promotes coherent and effective instruction within their school and even across their district.

Juli K. Dixon
Professor, Mathematics Education
School of Teacher Education
College of Community Innovation and Education
University of Central Florida

The Math Pact is an essential resource for educators looking to develop and support a coherent schoolwide approach to the teaching and learning of mathematics. The authors provide clear guidelines with examples and references to numerous resources that will support you and your colleagues as you develop a shared vision for you and your students in mathematics education.

Mike Flynn
Director, Mathematics Leadership Programs
Mount Holyoke College
South Hadley, Massachusetts

This book brilliantly connects research-informed practices to empower stakeholders in engaging students in meaningful mathematics through a vertically articulated Mathematics Whole School Agreement! Building on the impact of the *Rules That Expire* series, the authors lay out an easy to implement approach to share, connect, and represent mathematical ideas across classrooms to intentionally and explicitly bring about change prior, during, and after instruction.

Farshid Safi
Mathematics Education
School of Teacher Education
University of Central Florida

The Math Pact will surely be hailed a seminal work in the field for years to come! The authors outline a crystal-clear, research-supported case for a unified approach to mathematics instruction. The embedded reflection opportunities and suggested parent communications make a Mathematics Whole School Agreement possible for everyone.

Delise Andrews
3–5 Mathematics Coordinator
Lincoln Public Schools
Lincoln, Nebraska

Maya Angelou said, "Do the best you can until you know better. Then, when you know better, do better." *The Math Pact* is a practical guide that supports us in collectively taking responsibility for helping each and every one of our students to become problem solvers, critical thinkers, and capable and confident doers of mathematics. The detailed vignettes and suggestions are vivid guideposts for a journey of self-reflection and collective decision making with colleagues about really critical components of mathematics instruction. It will leave you both "knowing better" and collectively "doing better" for your students. No matter your level of experience, there is something new to learn here! I think it would work beautifully with pre-service teachers, new teachers, and veteran teachers. I even picked up some new things on my read that I hadn't really thought about in my nearly 30 years in the classroom.

Shawn Towle
Mathematics Teacher
Falmouth Middle School
Past President
Association of Teachers of Mathematics in Maine

Wow! *The Math Pact* will lead the movement to help educators and students overcome the idea that math is a mysterious set of "tricks and shortcuts." The Mathematics Whole School Agreement process provides the steps, language, representations, and knowledge to build, implement, and sustain equitable learning outcomes for all students! The power to make change is in our collective hands and hearts! This book needs to be in the hands of all teachers, district leaders, and stakeholders.

Cathery Yeh
Assistant Professor
Attallah College of Educational Studies
Chapman University

This is a long-awaited publication that will help preservice teachers, educators, and administrators of all levels and curriculum coordinators abolish the use of tricks and magic in mathematics instruction. For years we have inadvertently led students down a dead-end street in their math instruction by teaching them expiring rules, tricks, and cutesy sayings that may help them perform short-term on a test but leave them conceptually damaged in the long run.

Julie Duford
Fifth-Grade Teacher
Polson Middle School
Polson, Montana

This is the perfect balance of inspiration and practical guidance! The inspiration motivates me to work harder at collaboration with peers, building common commitment. The practical guidance helps me put the ideas into action around what specific changes will improve mathematics teaching and learning.

Lynn Selking
Mathematics Consultant
Great Prairie AEA
Wapello, Iowa

The Math Pact
The Book at a Glance

Consider this book your handbook and go-to guide for ensuring equitable, coherent instruction across grades, schools, and your district. You'll find a number of features throughout the book to aid you in your journey creating a Mathematics Whole School Agreement (MWSA).

FIGURE 2.1 • WORDS THAT EXPIRE IN ELEMENTARY SCHOOL

Words that expire	Expiration details	MWSA-suggested alternatives
General		
"Show your steps"	"Show your steps" suggests that the student should be carrying out a procedure.	Instead, we recommend saying "Explain your thinking," as this phrase is inclusive of multiple options of the possible mathematical representations (e.g., concrete models, illustrations, words, graphs, symbols) and multiple strategy options.
Numbers		
Using the words *take away* as the generic way to read a subtraction sign in an equation—such as 14 – 8, read as "14 take away 8"	Not all subtraction problems are take-away situations and thus should not always be read that way.	Instead, simply use *minus* when reading such an expression or equation. Other ways to describe it include "14 subtract 8," "8 less than 14," or "the difference between 14 and 8."
Calling zero a *placeholder*	A placeholder is something that stands for something else. Zero is not a placeholder for another number.	Zero is a *number*, and as such it is a value that may in some cases represent no units or no tens, no tenths, no hundreds, no hundredths, and so on in the decimal representation of the number.
Reading a multidigit whole number such as 123 as either "one, two, three" or "one hundred *and* twenty-three"	Reading a number by its digits only does not promote understanding of the number's magnitude. When the word *and* is inserted, it implies that the number consists of a whole and a part, as in a decimal or fraction.	123 should be read as "one hundred twenty-three." The same is true for other multidigit whole numbers—no *and*. Meaning must be developed from the start, and there is no place value meaning given by calling out digits. However, the word *and* can be stated when you are reading a number that has a decimal point (as in 2.45 being read as "two *and* forty-five hundredths" or \$9.26 as "nine dollars *and* twenty-six cents") or a mixed number such as $3\frac{1}{2}$, read as "three *and* one half." When people in the media read a multidigit whole number and say, for example, for the year 2021, "twenty, twenty-one" or "two thousand *and* twenty-one," we hope your students catch those and say "No *and*!"
	Bigger and *smaller* are	

In-depth charts will help you find a consistent approach to preferred and precise mathematical language, notation, representations, rules, and generalizations that will help clarify students' mathematics understanding.

FIGURE 5.3 • RULES THAT EXPIRE COMMONLY USED IN ELEMENTARY SCHOOL AND SUGGESTED ALTERNATIVES

Rule that expires	Expiration details	Suggested alternatives
Numbers and operations and algebraic thinking		
Addition makes numbers bigger, or you should always expect a larger answer when you add.	When students begin learning about the operations of addition, they are often given this rule as a means to develop a generalization relative to operation sense. However, the rule has many counterexamples—some immediate. For example, addition with zero does not generate a sum larger than either addend. It is also untrue when adding two negative numbers (e.g., $-3 + -2 = -5$), because -5 is less than both addends.	The main focus should be on teaching that the meaning of addition involves combining or joining quantities. Students should talk about the reasonableness of their answers, and giving them "fake" student work where mistakes can be discussed is far better than just giving them a rule that "bigger" answers are expected and that condition alone means that they have reasonable answers.
You cannot take away a larger number from a smaller number.	Students may first hear this phrase as they learn to subtract whole numbers. When students are restricted to only the set of whole numbers, subtracting a larger number from a smaller one results in a negative number—an integer that is not in the set of whole numbers—so this rule is true. But later, when students encounter applications or word problems involving contexts that include integers, they learn that this "rule" is not true for all problems.	In the early years, this rule can cause a great deal of confusion. When students are subtracting two-digit numbers as they think of this rule they might give this response: $\begin{array}{r} 32 \\ -15 \\ \hline 23 \end{array}$ The preferred way to present this idea is to use concrete materials—such as base 10 materials for the double-digit subtraction problem above. Start by representing the minuend with the materials; so in this case you would show 3 tens and 2 ones (preferably on a place value mat). Ask a student to remove 5 ones. Hopefully, they will not say you cannot take away a larger number from a smaller and instead say, "I can't do that." Ask them to consider the entire quantity of 32—can you

Remember, as you work through this chapter, you're actively establishing the RTE component of your MWSA—you're making great progress!

WHAT ARE RTEs?

Rules that expire: Tricks, shortcuts, or rules that are used in mathematics that immediately or later fall apart or do not promote mathematical understanding.

RTEs are a deeply rooted tradition in mathematics education, a means to teach a procedure or a strategy in a way that the teacher believes makes the learning easy and fast or helps students remember. Sometimes RTEs are used with the best of intentions as an attempt to make learning "fun." However, let's be clear: RTEs are harmful in the long term and should not be used. We authors learned this the hard way by teaching these rules in our classrooms only to regret it later when we taught other grades or learned more mathematics content. RTEs might temporarily seem to help in the short run, but in the long run they support the myth that mathematics is a set of disconnected tricks and shortcuts, is magical, or at worst is incomprehensible. The basic premise of RTEs is to teach for convenience or speed, and the subsequent initial appearance of student success fuels the continuance of teaching these rules. In other words, being able to apply RTEs by rote may get students through the next problem, quiz, test, or high-stakes assessment, making it seem as though there is deep conceptual understanding (or a strong reason to teach this way) when often there is not. Then, when that appearance of success leads us to believe that students understand more than they do, we use the RTEs again. In essence, the use of the "trick" or the "shortcut" becomes a self-fulfilling prophecy. Instead, we should teach for the future mathematics we know is coming and emphasize enduring understanding and

CORE MWSA IDEA

Even actions we take as teachers that seem well-meaning can be harmful in the long run!

CORE MWSA IDEA

Throughout the book, find definitions of key terms and notes on core MWSA ideas.

REFLECTION

CONSTRUCTION ZONE—WHAT REPRESENTATIONS ARE MOST BENEFICIAL AND SPAN THE GRADES?

As you think about the representations you will use as part of your MWSA, consider these questions:

- Which representations can you agree on that will span multiple grades?
- Which representations have you used that are not productive in terms of helping students learn or for which you may not know all the options for using them?
- Which representations might cause confusion or create or perpetuate misconceptions?

Using the following space, record representations that are being used that need to be rethought, those that might need further explanation, and others that can and should be used across the grades. Then, as you continue reading this chapter, other suggestions may help you spark new ideas or prompt you to reconsider what can be used as appropriate alternatives.

Reflection tasks help you consider how key ideas relate to your own instruction.

TRY IT OUT

MWSA HANDOUT FOR REPRESENTATIONS

Representations We Are Using in ____

Representations that may cause confusion	Agreed-on representations in our whole school agreement

Try It Out and Things to Do sections provide concrete opportunities to directly engage with your team in creating a Mathematics Whole School Agreement.

THINGS TO DO

Send the Letter

Hello ___________________________,

We have already written to you about the Mathematics Whole School Agreement (MWSA) that we are developing across the entire school this year. As you know, we are all working hard to align our instruction in mathematics across the grades. As you may remember, earlier this year you received a letter where we talked about the mathematical language and notation we use during instruction. We are now looking at the representations we use in mathematics. As a mathematics team, we have agreed on the physical materials we may use to model the mathematics and the ways in which we explain the mathematics by means of pictures or diagrams and mathematical symbols. Everyone in the school involved in the teaching and learning of mathematics is using these and is focused on teaching for students' depth of understanding and connection to mathematical ideas within and across grades. The way we model in mathematics has an effect on the way students understand mathematical ideas. We want your student to become an adult who knows mathematics and will succeed in whatever they choose to do in life. We thank you for joining us in making this shift to be consistent in how we support your student as we prepare them for their personal and professional future.

Thank you for your help,

Your student's teachers and principal and members of the school community

the MATH PACT

ELEMENTARY

the MATH PACT

Achieving Instructional Coherence Within and Across Grades

ELEMENTARY

Featuring Rules That Expire and Other Dos and Don'ts

KAREN S. KARP
BARBARA J. DOUGHERTY
SARAH B. BUSH

Foreword by Robert Q. Berry III and Matt Larson

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CONTENTS

online resources

Visit the companion website at
resources.corwin.com/mathpact-elementary
for downloadable resources.

FOREWORD

As educators, we often focus on the observable learning differentials between countries, states, and school districts, and between schools within a single district. With great clarity, Karp, Dougherty, and Bush demonstrate that in focusing on these differentials we are missing what is often a more significant differential—the differences in learning outcomes that exist within schools between teachers of the same grade level or subject.

Connected to and building on the latest mathematics education literature, the authors argue that it is essential that all stakeholders within a school collaboratively agree on and commit to following a Mathematics Whole School Agreement (MWSA). By making such a commitment, each school community can approach mathematics instruction in a unified and consistent manner. The fact that the authors recommend a *whole school* agreement is significant. While many schools today are engaged in professional learning community work, those communities, when effective, may only address horizontal consistency within a grade level or subject. Effective professional learning communities within an MWSA help ensure the needed vertical consistency in addition to horizontal consistency.

Even when the same curriculum and standards are used schoolwide, the outcomes students experience in different classrooms can vary greatly due to inconsistencies in notation, language, representations, instructional strategies, assessment techniques, depth of learning, and the "rules" students learn in different classrooms. It is this lack of horizontal and vertical consistency that contributes to inequitable learning outcomes in American schools. At its heart, achieving more equitable outcomes is the goal of the MWSA, and the authors provide a process to build, implement, and sustain this necessary agreement in a school, and ultimately a district.

An MWSA provides three levels of benefits to achieve necessary consistency: teacher, student, and school levels. Teacher-level benefits include support for high-quality mathematics instruction, enhanced teacher learning, increased professional communication, reduced personal isolation, and closer alignment between curriculum and assessment. Student-level benefits focus primarily on increased student success on outcomes and depth in students' mathematical understanding, which positively influence students' mathematical identity and agency. School-level benefits include a positive influence on school climate, support for innovation, a cultural shift that emphasizes equitable opportunity and outcomes, schoolwide attention on the needs of students, flattening of the power structure, and fostering of a professional culture of intellectual inquiry.

As the authors state, "An MWSA must be grounded in a schoolwide commitment to equitable and high-quality mathematics instruction." The benefits of an MWSA address access and equity by supporting stakeholders' knowledge of the promises and challenges of the students they serve, providing a sense of collaboration for addressing potential obstacles that may limit access to high-quality mathematics teaching, and creating the space and sense of community necessary for stakeholders to ensure that the allocation of human and material resources is equitably distributed and meets the needs of both teachers and students.

In a school with an MWSA, the mathematical identity and success of each and every student become the collective responsibility of every adult involved in students' learning. We encourage you to take advantage of the authors' recommendations, collaboratively build an MWSA, make a commitment to its implementation, and make a difference in the learning outcomes of the students in your school and district.

Robert Q. Berry III, University of Virginia, Charlottesville
Past president, National Council of Teachers of Mathematics

Matt Larson, Lincoln Public Schools, Nebraska
Past president, National Council of Teachers of Mathematics

PREFACE

WHAT IS THIS BOOK ABOUT?

Imagine teaching at a school where mathematics instruction is coherent, high quality, and consistent across classrooms and grade levels. No matter which teacher a child has, they are receiving the highest-quality mathematics learning experience. All teachers in the school are working together as a team, a true team that considers the success of each and every student in the entire school as a collective responsibility. As students progress through the grades and have different teachers, they see how mathematical ideas connect, and they use familiar representations and consistent and appropriate mathematical vocabulary and notation. Teaching is done in a way that develops deep mathematical understanding, and the team knows that taking more time up front to develop concepts, connections, and procedural fluency will pay off in the long run, even saving time. Both teachers and students are excited by and feel empowered by mathematics. Welcome to *The Math Pact*, where you are about to embark on creating a Mathematics Whole School Agreement (MWSA)!

WHO IS THIS BOOK FOR?

If you are a teacher of mathematics, mathematics instructional coach, curriculum leader, principal, special education teacher, paraprofessional, parent, tutor, or preschool teacher, or anyone involved in ensuring that children are successful in mathematics, we are calling your name to join this movement toward unity of message and purposeful alignment of best practices. This book is for you and the children you teach. In short, if you care about doing what is best for students, this book *is* for you!

OUR UNIQUE AND INNOVATIVE APPROACH

Actually, our approach is really just common sense. It's all about getting everyone on the same page. We are not suggesting losing the individual style teachers have or eliminating the magic of their personality infused into mathematics instruction; we are talking about best practices and precision. These are not points of academic freedom; rather, they are ways to work toward the best interests and learning of mathematics by children in preparation for the adults they will someday be, through the implementation of research-informed best practices.

WHAT INSPIRED US

Years ago Karen started talking in her presentations at National Council of Teachers of Mathematics (NCTM) conferences and institutes about the need to avoid rules that expire (if you are not sure what these rules are, keep reading). With the encouragement of Sarah and Barbara, the three formed a team to write first an article for *Teaching Children Mathematics*, followed by other grade band articles in *Mathematics Teaching in the Middle School* and *Mathematics Teacher*. Two of these articles are in the top 10 downloaded journal articles (most recent figures) published by the NCTM: "12 Math Rules That Expire in the Middle Grades" (ranked #1) and "13 Rules That Expire!" (ranked #8). In addition, "13 Rules That Expire!" was selected as the NCTM Editorial Pick of the Year for *Teaching Children Mathematics* (2015) and was reprinted in 2019 in the compilation journal, *The Best of Teaching Children Mathematics, Mathematics Teaching in the Middle School, and Mathematics Teacher on Questions, Discourse, and Evidence*. This response was very encouraging. Many people contacted us by email to say that they resonated with the very things we had learned over the years. They wanted these ideas faster than the way we learned them, which was very slowly over the years (sorry to all our former students!)—they wanted them *now*. The next step was logically to bridge these ideas across the grades, so we wrote an article about establishing the whole school agreement, which appeared in *Teaching Children Mathematics* in 2016. After these published pieces and approximately 15 presentations at conference venues and many talks in school districts, the grade-level books seemed the next logical step to share these ideas that teachers were emailing us about, tweeting us about, discussing in their own articles and citing us, and sharing in their presentations. We thank everyone for the encouragement—it led to this series.

WHAT'S IN THE BOOK, AND HOW CAN YOU USE IT?

Consider this book your handbook and go-to guide for ensuring equitable, coherent instruction across grades, schools, and your district. This book is organized into three parts. In Chapter 1, we provide an introduction that includes describing what an MWSA is and why it is critical to the success and well-being of each and every student. In Chapters 2–6 we dive into each component of an MWSA, providing detailed vignettes and suggestions as you consider and develop your own MWSA as a team. Finally, in Chapters 7–9 we delve into the enactment of an MWSA, including incorporating it into all of your team's lessons, across your school and district, and we share success stories from those who have made this work a reality, transforming the teaching and learning of mathematics in their setting.

We hope you too feel the urgency for an agreement as a way to promote students' learning of mathematics. By providing examples and stories of our own missteps and recovery, we are hopeful we can help you navigate around our errors. Via a step-by-step walk-through of the process, we want to support you in your teamwork and your personal work. Along the way we try to point to timely resources embedded in the chapters and share why this is the right approach. We recognize that this book can never address every aspect you will need to consider as you craft your own MWSA. The purpose of this book is to provide a foundation, and then you can build off this work, using additional resources to best meet the needs of each and every student in your setting.

INSPIRATION TO JUMP-START YOUR WORK

Collaboration can save time rather than suck up time if people work productively toward common goals. The power to make change is in your hands and in your heart. It takes both hands and heart to work through the decisions that must be considered in the activities and reflections in this book that will guide your thinking. Chapter by chapter we will unveil a twofold process—one that is founded on the forging of a team and the other a self-guided and self-empowered learning opportunity. Pause and reflect, but do make change in support of having all students experience the joy, wonder, and lifetime usefulness of mathematical understanding.

ACKNOWLEDGMENTS

Our goal is to provide support for all learners by ensuring that students have a seamless and cohesive mathematical learning experience, where mathematical language, notation, representations, rules, and generalizations flow from grade to grade. Embarking on a Mathematics Whole School Agreement (MWSA) is a collaborative and exciting endeavor. This book would not have been possible if it weren't for the many partners and collaborators we've had along the way. They have been the inspiration for this work.

We wish to thank Bob Ronau, who created several of the figures for this book. A special thank you to Richard Cox, Megan Wise, Angela Torpey, Erin Russo, and the teachers and administrators of the Discovery School, Old Mill School, and Sigsbee Charter School. We also wish to thank those who contributed to this book but wish to remain anonymous. These are the stories that make our work worthwhile.

We want to thank the fans of the MWSA who work across North America, who stay in contact with us and provide heartwarming stories of how they have become revitalized by the act of the agreement process and the establishment of cohesiveness.

We are very thankful to our publisher at Corwin, Erin Null. She would come to our sessions at the conferences and share how important she felt these ideas were for mathematics education. She loved that we were asking schools to seek harmony over many significant mathematics teaching components, such as language and rules. Her own experiences as a parent and her thoughtful comments helped us look at our approach with a new lens. We are grateful for her keen insights and enduring support.

Thanks must also be extended to Jessica Vidal for orchestrating the structure of the project and for translating the family letters and activities into Spanish. Additional thanks go to the entire Corwin team and everyone involved in our book's production, which includes Caroline Timmings, Tori Mirsadjadi, Talia Greenberg, Gail Buschman, Margaret O'Connor, as well as QuADS Prepress and Integra.

We would also like to thank Robert Q. Berry III and Matt Larson, both influential past presidents of the National Council of Teachers of Mathematics, for writing the foreword for this book. Their background of national leadership roles in K–12 education with a focus on equity linked them logically to this project. We are grateful for their support and their insights into this work.

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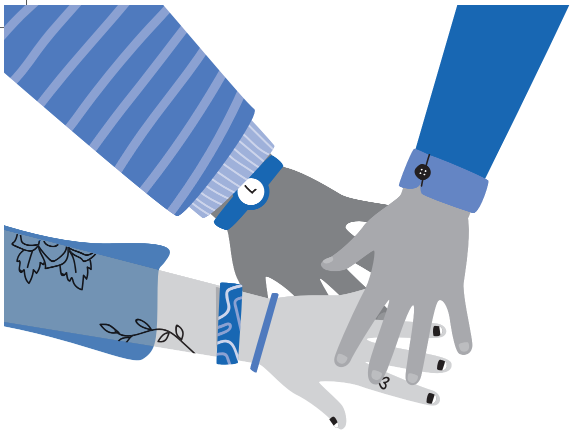
Barbara J. Dougherty is the director of the Curriculum Research & Development Group and a professor in the College of Education at the University of Hawai'i. She is a former member of the board of directors of the National Council of Teachers of Mathematics. She serves on the author panel for the *What Works Clearinghouse Practice Guide* on assisting elementary school students who have difficulty learning mathematics for the U.S. Department of Education Institute of Educational Sciences. She is the author or coauthor of approximately 22 book chapters, 28 articles, and 35 books, including *B*[Hold] *Explorations to Promote Algebraic Reasoning*. She also served as the series editor for the NCTM series *Putting Essential Understandings Into Practice* and grade-band editor for *Essential Understanding for Teaching and Learning*. Her research, funded by more than $11.5 million in grants, emphasizes supporting students who struggle in middle and high school, with a focus on algebra. She holds teaching certifications in middle and high school mathematics and K–12 special education.

Sarah B. Bush is an associate professor of K–12 STEM education and the program coordinator of the mathematics education track of the PhD in education at the University of Central Florida in Orlando. She received her doctorate in curriculum and instruction with a specialization in mathematics education from the University of Louisville. Since 2010 Dr. Bush's productivity includes more than $4.8 million in externally funded projects, nine peer-reviewed published books, more than 70 peer-reviewed publications, and more than 100 peer-reviewed and invited international, national, regional, and state presentations. She is actively involved in the National Council of Teachers of Mathematics (NCTM), currently serving as an elected member of the board of directors (2019–2022). Dr. Bush was the lead writer of NCTM's *Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations*, published in 2020. Her scholarship and research focus on deepening student and teacher understanding of mathematics through transdisciplinary STE(A)M problem-based inquiry and mathematics, science, and STE(A)M education professional development effectiveness. She holds a teaching certification in 5–12 mathematics.



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JUMPING ON BOARD

What Is the Mathematics Whole School Agreement?

Have you ever walked through classrooms in your school and looked at the items on the wall related to mathematics? Give it a try sometime, and consider what is similar and what is different across classrooms. What do you notice and wonder about? Perhaps you'll see a "Steps to Problem Solving" poster in your neighboring fourth-grade class and notice that they are using different steps from those in the poster in your classroom. Or maybe you'll see that two different first-grade classrooms have displays of possible mathematics thinking strategies on the wall but they don't match. You may see math word walls with completely different names for mathematical properties or algorithms. What, you wonder, will happen when those children move into second grade next year but their prior mathematical knowledge is substantially different? What confusion will ensue? How will the next year's teacher cope? What if that teacher is you? Or what if your job is to coach and support that teacher?

This book is designed to keep you, your colleagues, and your students away from this unfortunate, but all too common, situation.

In this chapter you will learn

- What a Mathematics Whole School Agreement is
- Why students need a cohesive mathematics instructional experience
- How equitable and high-quality instruction is at the foundation of the process

WHAT IS THE MATHEMATICS WHOLE SCHOOL AGREEMENT?

Mathematics Whole School Agreement: A unified and consistent approach to mathematical language, notation, representations, rules, and generalizations.

In this book we argue for the idea of building a Mathematics Whole School Agreement (MWSA). This initiative refers to a unified and consistent approach to preferred and precise mathematical language, notation, representations, rules, and generalizations that will help clarify rather than muddy children's mathematics understanding and increase their chances of mathematical success as they move into middle grades, high school, and beyond. In this book, we describe the need for an MWSA; we discuss what the agreement entails, including some very concrete mathematical don'ts and dos; and we share ideas about how to go about establishing and building the coordination and buy-in needed from educators and stakeholders to enact, implement, and get the best results from the MWSA.

So why the MWSA, and why now? *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (National Council of Teachers of Mathematics [NCTM], 2020) describes the need to broaden the purposes of learning mathematics and articulates three key purposes for learning mathematics in the early years:

- Develop deep mathematical understanding as confident and capable learners
- Understand and critique the world through mathematics
- Experience the wonder, joy, and beauty of mathematics (p. 11)

These three purposes of learning mathematics embody the essence of the mathematical learning experiences we most want for our students—all of our students. They empower students as mathematical thinkers and doers, and they prepare students with the mathematical literacy needed for their professional and personal lives (NCTM, 2020). An MWSA builds the instructional foundation needed for these key purposes of learning mathematics to be realized in a way that is consistent, coherent, systemic, and systematic within grades, across the school, and, more broadly, within a district, state, or province. Establishing an MWSA ensures that each and every student has access to mathematically sound, consistent, high-quality learning experiences. What might happen if we don't establish an MWSA? Let's peek into a classroom:

A third-grade teacher, Ms. Jackson, is engaging her students in several problems about multiplication situations using an equal-sized group model. The first problem asks the students to think about how many children could fit in the school library if there were four tables with three children at each table. The second problem asks the students to think about how many children could fit in the same library if there were three tables with four children at each table.

Ms. Jackson's students are familiar with this problem type and select manipulatives from a central basket on the table, with most children choosing two-colored counters to represent the children and paper plates to represent the library tables. During small-group work time, this conversation occurs:

Robin: I'm looking at these equations we wrote ($4 \times 3 = 12$ and $3 \times 4 = 12$), and I think this is the flip-flop property.

Ms. Jackson: I'm not sure of the flip-flop property; can you tell me more?

Robin: It's what we had in first grade. You can flip the numbers.

Jorge: Oh I know that one; my teacher last year called that the commuter property! She said commuters go back and forth to work and the numbers go back and forth.

Ms. Jackson: Robin, can you show me how the flip-flop property works with an example?

Robin: Sure, $2 + 3 = 5$ and $3 + 2 = 5$. You can flip them or flop them, and you get the same answer.

Ms. Jackson: Jorge, what about you? Can you give me an example?

Jorge: Yes. It's like Robin said. If you have four children at three tables and switch them back and forth, like a commuter, you can also put three children at four tables. See? [*He rearranges his counters once and then back again*.] I know this because my dad commutes, so he goes back and forth.

Ms. Jackson: I see. I think you are both talking about the commutative property. You are right in thinking there is something similar happening here with multiplication as there was with addition. Let's put "Commutative Property" on our math word wall with the equations you pointed out, Robin, so we can all use it when we notice this property appearing at other times during mathematics class.

This may seem like an extreme or even trivial example, but the dialogue from Ms. Jackson's class happens when students have at some point engaged in mathematics instruction where they are taught in ways that are inconsistent with what other teachers are using, are not well matched to the curriculum or standards, do not represent appropriate mathematical terminology, or suggest rules that later expire or fall apart. Have you ever seen this? The problem is that when consistent and appropriate mathematical language is not intentionally used, there is no evidence of vertical coherence.

Vertical coherence: The act of ensuring that interrelated mathematics concepts are aligned across grades.

Horizontal coherence: Being mindful of the relationship among mathematics concepts at the same grade.

That is, in successive grade levels, teachers and other students do not have a shared vocabulary or a shared understanding of how and when imprecise words are used. Even when the same curriculum and standards are used schoolwide, without intentional planning about what will be taught and how, the outcomes can be disjointed and students can become confused. Students begin to feel as though they're constantly learning something new and different. The irony is that while many schools work hard to enforce a unified approach to other educational matters across the school, the same is rarely true of mathematics instruction. Take classroom management, for example, where there are set guidelines for how students are expected to behave in classroom and schoolwide situations. School leaders and teachers wouldn't think to allow such inconsistency. Instead, they set out rules and norms for movement around the room or hallways between periods, when conversation is permitted, how to ask for help when you are not sure what you should do, and how to participate in discussions. These are agreed-on expectations that are consistent schoolwide. But we need to ask ourselves why there shouldn't be a similarly consistent agreement in place for teaching content. How much do discrepancies—and in some cases outright contradictions—in the way we teach mathematics and the words we use (e.g., *flip-flop property*) get in the way of having a coherent, high-quality mathematics program? How does this confuse and harm rather than help our children in their mathematical learning and achievement? How can we do better by our kids?

WHY STUDENTS NEED A COHESIVE APPROACH TO INSTRUCTION

The consistency of a message is important. We all know the feeling of having different people tell us different ways in which we need to do something and finding that hard to negotiate or navigate. Multiple communications to students with conflicting language and notation, representations, and rules and conventions in mathematics can cause mental conflict and stress for adults and children alike. This perpetuates the negative stereotypes about mathematics we hear so often: It isn't relevant to students' lives outside school; it's boring; it requires a "math brain"; it consists of a set of "disconnected ideas." To build a cohesive approach, we want to "maximize strategies that promote positive emotion" and diminish stress or threats that impede learning (Hardiman, 2011, n.p.). Research on brain-targeted teaching helps us understand how students sort the information they receive into whether those new pieces of data relate to prior experiences or knowledge

(Hardiman, 2012). Then the students build new ideas from there. If the information is in opposition to previous learning, there is a disconnect that can hinder learning or result in a backward step in retention of mathematics understanding. Squire (2004) suggests that how well we remember hinges on rehearsing and restating the ideas we learn as we set them into cohesive and connected long-term systems, constructing one layer of concepts on another. That can't happen if we don't present content in ways that help students find the familiar, identify patterns, and explicitly point out the connections between prior knowledge and new information (Skemp, 1978). Students need these linkages to deeply examine mathematics concepts and analyze situations through inductive problem-solving approaches rather than a strictly deductive model.

Deductive problem-solving: Where there is a presentation of information that students apply to the models they are taught. It is teacher centered.

Inductive problem-solving: Where students experience a more inquiry-based approach. It is learner centered.

HOW DOES AN MWSA PROVIDE A SOLUTION?

The MWSA's design moves away from fragmented approaches and a patchwork of instructional language and notation, representations, rules and conventions, generalizations, and problem-solving approaches across multiple grades to channel an effort toward desired goals and objectives shared by all. It offers the consistency students need because it

- is an agreement shared by *all* stakeholders,
- helps students make sense of the content, and
- helps teachers ensure alignment to the standards and assessments for which they are accountable.

An MWSA Is an Agreement Among *All* Stakeholders

The MWSA is grounded in the idea that students learn mathematics more deeply and successfully when the school has a plan that *all education stakeholders who engage with students* know and follow. All of these stakeholders need to be aware of and ready to implement what educators in the school or district agree on the specific language and notation, representations, rules and conventions, generalizations, and overall problem-solving approaches that every educator in the building or district will use (Karp et al., 2016). This process of reaching an MWSA purposely brings together a broadly defined team of stakeholders that not only includes teachers, instructional coaches, paraprofessionals, and administrators but also involves substitute teachers, volunteers such as grandparents and other local community members, student teachers, staff, all family members, and others involved in students' learning of mathematics. By following an MWSA

CORE MWSA IDEA

An MWSA must involve any and all stakeholders who participate in students' mathematics learning. teaching mathematics in a school is a team sport!

approach, the focus shifts to communicating as a unified whole about the discipline of mathematics and how it is best learned using research-informed practices. Without a clear agreement that is shared by the community as a whole, the result will be that every year the teaching of mathematics becomes harder and harder as students progress up the grades through different teachers and learning becomes more difficult for all students. Let's end this.

An MWSA Helps Students Make Sense of the Content

Some administrators and instructional leaders may say, "But we all have the same curriculum—doesn't that count?" And we respond, "That's a great start." (Later in this chapter we talk about schools that do not have a shared curriculum.) When teachers teach the same mathematics content and practices but use completely different instructional resources, the quality of mathematics instruction students receive will likely vary greatly and there is a strong risk of mathematics not being taught in coherent or consistent ways. This can occur both when teachers have a common curriculum but implement it very differently and when teachers do not have a common curriculum. These disjointed approaches lead to situations such as these:

- Teachers in subsequent grades believing that their students have prior mathematical knowledge that they do not possess
- Students harboring notions of disconnected mathematical relationships with gaps in conceptual continuity
- A general absence of the sense-making we'd like to develop in mathematics, which causes children to become confused and potentially dislike mathematics
- Students developing the feeling that they are not good at mathematics because what they were taught no longer holds true

CORE MWSA IDEA

Curricular coherence isn't about teachers teaching just what they know or sharing a collection of favorite activities!

No well-informed democratic society can afford that! Curricular coherence is about developing a consistent learning pathway in the school; it isn't about teachers teaching just what they know or sharing a collection of favorite activities.

An MWSA Helps Teachers Align Their Teaching With Standards and Assessments

Curriculum is different from, but informed by, the standards adopted in your setting. Although some states have officially adopted the Common Core State Standards in Mathematics (National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010), other states use what Opfer et al. (2016) refer to as Standards Adapted From the Common Core, and some others may use different state, provincial, district, or school standards. Regardless of the standards used, there remains much more to consider in an MWSA. In fact, there is little evidence of how standards are connected to what teachers actually do in their classrooms (Opfer et al., 2016). Standards documents themselves state that "standards establish what students need to learn, but do not dictate how teachers should teach. Instead, schools and teachers decide how best to help students acquire the content represented in the standards" (Common Core State Standards Initiative, 2016, n.p.). They go on to say, "Standards are not curricula and do not mandate the use of any particular curricula" (Common Core State Standards Initiative, 2016, n.p.). These statements are helpful because they not only honor teachers' critical function in decision-making but also expose the potential for using instructional approaches that lead to a disjointed collection of lessons. While teachers should feel empowered in determining their mathematics instruction, the effort should be a collaborative one with an emphasis on consistency and alignment. The MWSA requires that you work with your team of schoolwide stakeholders to establish a collective practice and focus on teaching in such a way that standards are implemented with depth and coherence, and the content and associated instructional practices across grades are aligned with attention to vertical or horizontal coherence.

CORE MWSA IDEA

While teachers should feel empowered in determining their mathematics instruction, the effort should be a collaborative one with an emphasis on consistency and alignment.

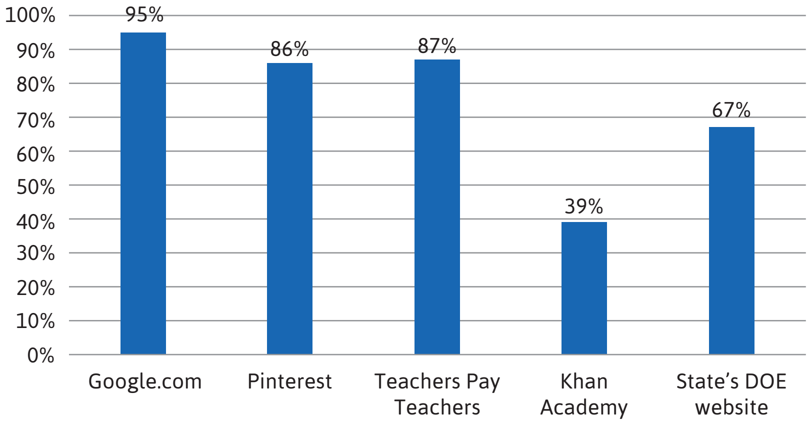
In asking teachers to know the mathematics content deeply and to effectively offer instruction to each and every student, it's important to acknowledge that many teachers are likely being asked to teach topics in ways they may never have experienced as a learner—either when they were in school or through their teacher preparation program. The difference is often more pronounced when we look at the mathematical practices (NGA Center for Best Practices & CCSSO, 2010) or the mathematical processes (NCTM, 2000), or other similar practices or processes adopted by your school, state, or province, because many teachers never experienced these sorts of standards when they were students. This challenge is compounded as some teachers are continuing to use more traditional instructional approaches to teach the rigorous ideas and concepts found in the required standards (Santelises & Dabrowski, 2015), which means that the standards may not be implemented as intended.

We (the authors) know what it is like to seek out curricular materials from near and far to help meet individual students' needs and to supplement content areas that need more attention. But searching for resources in the past often came with the luxury of sources that were well aligned with strong mathematical foundations and tended to be pointed to us via conference presentations, by colleagues who were master teachers of mathematics, or in NCTM journals where these resources and lessons were reviewed. They were often vetted through planning, analysis, implementation, reflection, and revision. In many cases, the experts were well versed in mathematics education and seen as more knowledgeable "others" who had based these resources on research or best practices. The resources were in some, but not all, cases reliably tested in classrooms, with solid results. Now the landscape is different and often involves nonvetted materials that don't always align with research, best practices, or standards. Additionally, the plethora of choices currently available feels like everyone is calling, "Look at what I think, or buy me." Kreisberg (2019) calls this freewheeling situation an "abundance of resources" (p. 1). She points to the enormous array immediately available at the click of a search term. But researchers (Iyengar & Lepper, 2000) suggest that sometimes, when what first appears to be alluring options becomes overwhelming, our decision-making can become seriously affected. This high number of choices can be debilitating when we become "too swamped to make meaning of them" (Kreisberg, 2018, p. 3).

Vetted resources: Instructional materials that have gone through a careful examination and rigorous review by an individual with expertise in that area.

Others are interested in exploring the effects of this smorgasbord of choices of instructional resources—such as researchers. The RAND Corporation has a standing interest in hearing from teachers in their well-known American Teacher Panel, a large group from across the United States whom they consult on a variety of issues. In one of their studies of 2,873 teachers, Opfer and colleagues (2016) found that 99% of elementary teachers said that they use materials "I developed and/or selected myself," and 96% of elementary teachers also reported that they use "materials developed and/or selected by my district." When asked about the use of resources found online, specifically the online resources they consulted most often, elementary teachers reported using, in order of frequency, google.com, Teachers Pay Teachers, Pinterest, their state's Department of Education website, and Khan Academy (see Figure 1.1).

FIGURE 1.1 • MOST POPULAR ONLINE RESOURCES REPORTED BY ELEMENTARY TEACHERS IN THE RAND STUDY

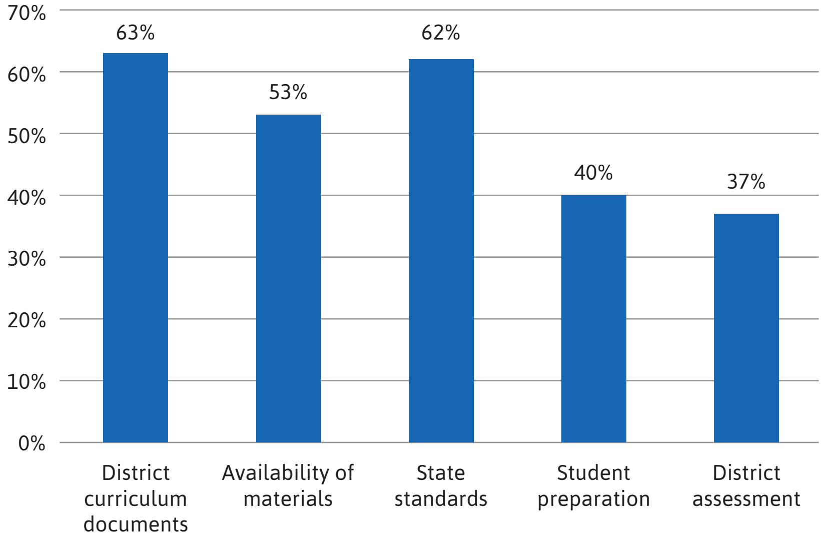


Source: Opfer et al. (2016, p. 39).

Note: DOE, Department of Education.

Interestingly, 57% of elementary teachers were required to use specific instructional materials, 27% said that materials at their school were recommended, and 15% reported having neither required nor recommended instructional materials in mathematics. It is clear from these data that teachers' use of self-selected or self-developed instructional materials is common. Furthermore, teachers reported that the factors that influenced their choices in mathematics instructional materials "a great deal" were district curriculum frameworks, maps, or guidelines; availability of materials; state mathematics standards; preparation of students for the next grade; and district mathematics assessment (see Figure 1.2; Opfer et al., 2016). Not surprisingly, they focused most frequently on the curriculum selected by the district and state standards.

FIGURE 1.2 • FACTORS THAT INFLUENCE TEACHERS' CHOICES IN SELECTING MATHEMATICS MATERIALS



Source: Opfer et al. (2016, p. 45).

When asked if their materials provide opportunities to engage in the use of mathematical language and symbols appropriately when communicating about mathematics, 56% of elementary teachers said "to a great extent" and 49% said that they teach major mathematics topics addressed by the state mathematics standards for their grade level coherently "to a great extent." In a nutshell, this also unfortunately means that 44% of teachers did *not* report using materials that use symbols and language appropriately and more than half of the teachers did *not* agree that they teach grade-level major mathematics topics addressed by state standards in a coherent way "to a great extent" (Opfer et al., 2016). We think you'll agree that this part of the findings isn't good news.

While many schools allow and encourage teachers to self-create or self-curate the curriculum by selecting from a variety of sources, this can result in some schools having different materials used in every classroom, even within the same grade, which isn't optimal. This practice is also not an equitable, coherent, or advisable approach. Please note that we are not talking about the need to address the different learning needs of specific students. We are talking about the core curriculum. Self-curated curriculum can inappropriately create qualitatively different learning experiences for students (as described in NCTM, 2020) and is not a good use of teachers' precious time. It also runs counter to the needed approach of teachers working as a collaborative team, which fosters their professional growth and collectively benefits students. A principal who was leading a middle school in such a situation described it as follows:

> The teachers know their kids well and what the students need to know. But if I look across the mathematics program, it is "hippity skippity." By "hippity skippity" I mean that teachers who don't follow a formal program can tend to be all over the place in their pacing calendar or choice of learning materials. They rely on their own understanding of what to teach and how to teach it, which may not reflect best practices or be grounded in a recommended, research-based learning sequence.

This principal made it a point to verify that all of his teachers are trying their best, but he acknowledged that some individual teachers' decisions about selecting materials had the potential to not align with the direction of the collective group and could be out of kilter with the vertical learning articulation across grades. Selecting materials in a piecemeal way can be chaotic and cause more effort to be put into a freelance approach, with everyone rowing in different directions, than the energy required of an MWSA, where everyone is rowing on a mathematics stream in unison. When many schools are relying on a curriculum in which components are selected or substituted with different replacements by different teachers, there needs to be a decided focus on what is nonnegotiable.

What does your school agree to say and do in the mathematics classroom? This resolution can be laid out in a nonnegotiable, strong, and unified way. For example, even if something that you have decided to avoid appears in a curriculum material, you remain resolute—you collectively won't say it and won't teach it (e.g., *reducing* fractions or a keyword strategy for solving word problems). Let's map out the route to reaching such an agreement.

REFLECTION

LET'S GET MINDFUL

- To what extent are children in your setting receiving the same qualitative mathematics learning experience? For example, to what extent are third-grade students being taught the same mathematical ideas in ways that are coherent and research informed? How are the third-grade teachers coordinating with second- and fourth-grade teachers? Or kindergarten and fifth-grade teachers?
- What are some mathematics instructional absolutes that teachers in your school (or district) must follow in unfaltering ways? What practices (e.g., lecture only, teaching as telling) should be avoided?
- What are some ways to build a cohesive team of stakeholders?

COMMITTING TO EQUITABLE AND HIGH-QUALITY MATHEMATICS INSTRUCTION

An MWSA must be grounded in a schoolwide commitment to equitable and high-quality mathematics instruction. In other words, if attempting to implement an MWSA in a setting where mathematics is taught in a procedure-driven, show-and-tell, lecture format, where there is only one way to get the one right answer, this is neither equitable nor high-quality instruction. A key part and benefit for all educators of the MWSA process is learning more deeply the *what* and *how* of engaging in equitable and high-quality mathematics instruction and embracing a shared commitment to aim for this ideal. Let's break down each element a bit.

Equitable Instruction

Equitable instruction: Classroom practices that ensure that each and every student has equitable access to challenging mathematics learning opportunities.

Equitable instruction includes a commitment to developing students' positive mathematical identities and strengthening their sense of mathematical agency. This means that each and every student is seen as mathematically competent and capable and they are empowered as mathematical thinkers and doers (NCTM, 2020). Aguirre et al. (2013) define a student's mathematical identity as the "dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways" (p. 14). In a classroom in which mathematical authority is shared, students are allowed time to form their ideas and think mathematically; they engage in meaningful discourse, and their contributions are valued (Berry, 2019). Equitable instruction in the elementary grades also attends to the unique needs of young learners while aggressively working to dismantle deficit views and adopt a strengths-based approach (as described in Kobett & Karp, 2020). As simply stated in *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (NCTM, 2020),

> We must openly challenge deficit labels and the institutional tools and practices that perpetuate static views about children's mathematical abilities and about who is or is not ready to learn. Each and every child is always ready and eager to learn more about their mathematical world. It is the adults that must reexamine their beliefs about readiness and learn to notice and support children's ever-evolving mathematical strengths. (p. 32)

Instructional practices can have both equitable and inequitable outcomes. Inequitable instructional practices will continue to privilege some students while marginalizing others. Establishing an MWSA is part of the hard work that must be done to make things equitable and just. An MWSA ensures that each and every student has foundational access to all of the mathematics opportunities they rightfully deserve.

High-Quality Mathematics Instruction

Planning for high-quality mathematics instruction should be wisely guided by NCTM's (2014a) eight mathematics teaching practices as first described in *Principles to Actions: Ensuring Mathematical Success for All* (see Figure 1.3). The eight mathematics teaching

FIGURE 1.3 • NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS TEACHING PRACTICES

Mathematics teaching practices
Establish mathematics goals to focus learning
Implement tasks that promote reasoning and problem-solving
Use and connect mathematical representations
Facilitate meaningful mathematical discourse
Pose purposeful questions
Build procedural fluency from conceptual understanding
Support productive struggle in learning mathematics
Elicit and use evidence of student thinking

Source: NCTM (2014a). Reprinted with permission from *Principles to actions: Ensuring mathematical success for all*, copyright 2014, by the National Council of Teachers of Mathematics. All rights reserved.

practices inherently represent effective, high-quality, student-centered instruction and should be at the foundation of any mathematics program establishing an MWSA. When these practices are implemented systemically, systematically, and equitably across a school, each and every student can have access to a high-quality mathematics program. To guide professional learning of the eight teaching practices in your school, *Taking Action: Implementing Effective Mathematics Teaching Practices in K–Grade 5* (Huinker & Bill, 2017) provides an in-depth discussion and examples from classrooms for each of the eight teaching practices.

PRIORITIZING THE DEVELOPMENT OF DEEP MATHEMATICAL UNDERSTANDING

An essential foundation for any MWSA is a commitment to developing students' deep mathematical understanding of both conceptual and procedural knowledge. Ensuring that students develop deep mathematical understanding requires a commitment to teaching in a way that builds procedural fluency from conceptual understanding (NCTM, 2014b). Students should be "doing mathematics" in ways that focus on (a) reasoning and sense-making, (b) the mathematical practices or processes adopted in your setting, and (c) grade-level college and career readiness standards. Students should be doing mathematics (as described in Smith & Stein, 1998) through the implementation of tasks that are cognitively rigorous and relevant, offer various solution approaches, and enhance students' sense-making of a variety of mathematical ideas. (For more information on developing students' deep mathematical understanding, we suggest reading Chapter 5 of *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* [NCTM, 2020].) Along the way in this book, you will likely find times when you and your team need to brush up on the content knowledge and pedagogical content knowledge (PCK) needed for teaching mathematics. We suggest exploring the grades PK–2 and 3–5 books from the two NCTM series *Developing Essential Understanding* (2010–2013; content focused) and *Putting Essential Understanding Into Practice* (2013–2019; PCK focused). An MWSA should be built around a schoolwide instructional plan that aligns with the professional commitment of all teachers of mathematics to developing students' deep mathematical understanding. Making this pledge means avoiding disjointed and surface-level changes (e.g., using consistent vocabulary but not engaging students in deep conceptual learning) that will ultimately not prepare children for their mathematical future.

CORE MWSA IDEA

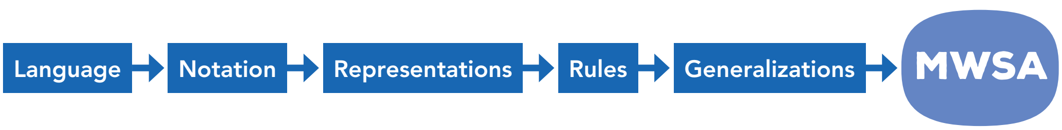
To change your practice, you have to practice change!

THE MWSA PROCESS

As we move to accept the thinking that change is not a passing fad that will simply disappear but, rather, something that benefits all players permanently, we will discuss two main components of the MWSA. First, we will detail the following central components of what all teachers and other stakeholders are agreeing to (see Figure 1.4):

- Correct and consistent language (Chapter 2)
- Precise notation (Chapter 3)
- Cohesive and consistent representations (Chapter 4)
- Evaluating rules that expire (RTEs; Chapter 5)
- Building generalizations and developing instructional strategies (Chapter 6)

FIGURE 1.4 • CENTRAL COMPONENTS OF AN MWSA JOURNEY



Note: MWSA = Mathematics Whole School Agreement.

Then, after you read Chapters 2–6, you'll be immersed in the second component of MWSA (Chapters 7–9), which is the last step in Figure 1.4 and an expansion of the agreement process discussed above, including everyone's commitment to it, their willingness to make change, effective instructional strategies, the structure of the lessons, and the eventual outreach to others. Not only does this process involve teamwork in structuring MWSA-aligned instruction, but you'll also explore next steps for expanding and refining this MWSA work and ensuring long-term sustainability.

In the following Reflection, predict what might be the easiest pieces for colleagues to agree to. How will exploring the next five chapters support your school as you consider developing an MWSA?

REFLECTION: MWSA–FORECAST

Think about the next five chapters, which will form the foundation of your MWSA. Here are some prompts to spark beginning discussions in your professional learning team or as coaches or mathematics leaders begin to think about implementing these ideas:

1. Who might you enlist as early adopters to help build your MWSA team?
2. What are some strategies you might use to gain buy-in from those who are initially resistant to the idea of an MWSA?
3. What do you think will be the easiest aspect of the MWSA for your school to agree on?
4. What are some potential challenges for both veteran teachers and novice teachers that you can predict?
5. How might the MWSA be integrated with your current curriculum materials in the school?
6. How might the MWSA lead to work that is more aligned with your content standards and mathematical practice or process standards?
7. What materials do you forecast you will need to implement the MWSA?

The following template will travel with you throughout the book. We show it here as a starting point to jot down notes as you move through the various chapters. What will you commit to in each component? Then you can partner with others and eventually discuss as a whole group what will go into your MWSA. Keep a copy of this form in your book, as it will serve as a reminder to answer the question "What will you commit to?" The more each person agrees to make changes, the stronger your agreement and your school, and your students' mathematical knowledge will be. Let's jump on board!

TRY IT OUT

Name: ______________________________ Grade: ______________

Language
Notation
Representations
Rules
Generalizations
Instructional strategies
Lesson structure

Source: Template inspired by Karp, K., Bush, S. B., & Dougherty, B. (2016).

online resources Available for download at **resources.corwin.com/mathpact-elementary**

PUTTING IT ALL TOGETHER!

In the book *The Multiplier Effect: Tapping the Genius Inside Our Schools* (Wiseman et al., 2013), the authors describe the characteristics of people who are either multipliers or diminishers. They suggest that when people take on the role of multipliers they can build the "collective, viral intelligence in organizations" (p. 19). Multipliers will try to implement the MWSA and gather together as a force all those who are engaged in teaching children mathematics, to build over time the strengths of each and every student and child. This approach of multiplying the talent of teachers "generates the collective will and stretch needed to undertake the most paramount of challenges" as they invest in a collectively agreed-on cause (Wiseman, 2017, p. 126). In this case the cause is developing mathematically literate members of a democratic society who are well positioned to make contributions to their communities and workplaces and who feel empowered to make the world a better place.

NEXT STEPS

Now that we've started on this journey, you are seeing the full landscape of the task ahead. What stands out to you about the MWSA? What surprises you? What makes sense to you and resonates with your teaching approach? What worries you? Who is the first person you will ask to join you on this quest? Continue this journey with us as we launch into establishing your MWSA with correct and consistent mathematical language in Chapter 2. We will investigate strategies for developing a common language and notation for the elementary grades. We will also consider how these beginning steps will shape the process you will use throughout the MWSA in getting your team talking about the mathematical ideas and solidifying the ways in which decisions will be made.