WHAT YOUR COLLEAGUES ARE SAYING . . .

"*Productive Math Struggle* will be a companion to all teachers seeking to enhance their math instruction, while supporting teachers to build the kind of math identities that our students deserve. There are so many gems within this book, it's hard to read without thinking of something to implement immediately into one's teaching practice."

Melissa J. Black
Associate Dean of Harlem Village Academies PEI Graduate School
Looping Second/Third-Grade Teacher at Two Rivers Public Charter School
Harlem Village Academy and Two Rivers Public Charter School
New York, NY

"Let's be honest. Creating a classroom environment that supports productive struggle is a challenge. This book will be your rescue story, packed with ready-to-use resources to support productive struggle and provide access and equity for *all* students to learn mathematics at high levels. This is for educators, coaches, administrators, and families to reflect on why productive struggle is imperative to success. Be courageous, take the first step, and embark on this journey of struggle! You won't regret it."

Tara Zuspan
Elementary Instructional Coach
Lincoln Public Schools, NE

"Teachers are expected to foster productive struggle in their students, and, *finally*, there is a comprehensive resource that lays out exactly how to make this happen! *Productive Math Struggle* is a playbook that includes six Actions—Value, Foster Identify, Build Community, Plan, Support, and Reflect—for ensuring teachers have the tools to help their students persevere in math class. This brilliant resource provides practical activities, lessons, ideas, and examples that can be used in the classroom on a daily basis."

Marissa Walsh
Elementary Math Instructional Coach
Blue Springs School District, MO

"Students need to feel successful during math. This book gives you tools to help your students build confidence with their understanding of mathematics concepts. It helps promote a love for math in a successful way by allowing them to process what they are learning."

Tamara Daugherty
Third-Grade Teacher, Zellwood Elementary
Lakeville, FL

"*Productive Math Struggle* moves us from *knowing we should* to *knowing how to* engage students in productive struggle. Through vignettes, showing ways to modify tasks, and dozens of instructional strategies, and engaging and helpful discussions, we can generalize actions that support and inhibit student thinking and learning. This book challenges us to challenge our students."

Jennifer Bay-Williams
Professor, University of Louisville
Pewee Valley, KY

"Providing opportunities to support productive struggle in the teaching and learning of mathematics is important. And, guess what? It's certainly time to do more than merely acknowledge such importance! This valuable and highly usable resource fully explores and establishes the considerations necessary to regularly provide and support learning opportunities that value and celebrate productive struggle in mathematics. Most importantly, this book recognizes and discusses what all teachers must think about and address as they challenge their students mathematically, all the while nurturing the importance of perseverance—a valued life trait. I expect this book to be a popular and valued reference for professional learning in the field of mathematics education for quite some time!"

Francis (Skip) Fennell
Past President, National Council of Teachers of Mathematics (NCTM)
Professor of Education and Graduate and Professional Studies, Emeritus
Project Director, Elementary Mathematics Specialists & Teacher Leaders Project
McDaniel College, MD

"The authors do a great job of defining what productive struggle is, as well as what it isn't. You can tell they are very familiar with classroom dynamics, and they offer helpful perspectives regarding a variety of issues: task modifications, issues to think about, student and teacher behaviors, productive and unproductive beliefs, as well community. The classroom vignettes were my favorite part because they helped me see what it looks like when students are doers of mathematics and teachers are supporting productive struggle. I loved the book!"

Jennifer Outzs
Middle School Math Teacher
NCTM Board of Directors
Pinellas County, FL

"*Productive Math Struggle: A 6-Point Action Plan for Fostering Perseverance* addresses the intricacies of productive struggle more than any other resource out there and is a must-read for every educator dealing with the challenges of implementing productive struggle in their classrooms. The authors offer an abundance of practical strategies to help you foster, build, and support productive struggle in your class. This book will be your go-to resource for everything about productive struggle, and I can't wait to use it in my work with teachers, coaches, and administrators."

Rosalba McFadden
Elementary Math Consultant
Zenned Math, Inc.
Monroe, NY

"I highly recommend *Productive Math Struggle: A 6-Point Action Plan for Fostering Perseverance* for all mathematics teachers, instructional leaders, and administrators as a guide to enact strategies that empower and rightfully position each and every student as thinkers and doers of mathematics. Read this book and be ready to be drawn in by powerful and relatable vignettes that illuminate what it means to support students in the deep learning of mathematics through productive struggle. With a valuable collection of practical prompts, activities, inventories, surveys, and reproducible tools, this book is a must-have as we work to prepare each and every student for their bright futures!"

Sarah B. Bush
NCTM Board of Directors
Associate Professor, K–12 STEM Education
Program Coordinator, PhD in Mathematics Education
University of Central Florida

"Although each of us, and each of our students, has experienced struggle, there is very little literature that helps us as mathematics teachers utilize and place a value on productive struggle. This book helps fill that void. *Productive Math Struggle* offers six essential actions for nurturing productive struggle before, during, and after our lessons to create a classroom and school community that embraces struggle in our students and makes productive struggle a natural part of the classroom community. This book gives every teacher and every school the tools needed to value and reward productive struggle in our mathematics classrooms."

David Ebert
Mathematics Teacher
NCTM Board Member, 2017–2020

"Engaging students in productive struggle is crucial for developing problem solvers with perseverance. *Productive Math Struggle* provides actionable changes for you to cultivate an environment where students have the opportunity to struggle. The authors suggest ways for you to develop and adapt tasks that are worthy of your students' efforts. Classroom vignettes offer contrasting points of view to help you make sense of how student struggle is supported by some teacher actions and inhibited by others."

Juli K. Dixon
DNA Math
Orlando, FL

"As educators, we have the privilege and crucial role of shaping students' math identities and encouraging productive struggle skills. This book will be an incredible, comprehensive resource as we continue to set a firm mathematical foundation for all students and lead them in their own journey of productive struggle."

Kristy Kennedy
Interventionist, Elementary School
Lincoln Public Schools, NE

"*Productive Math Struggle* synthesizes all the essential classroom practices for real mathematics learning! This resource provides teachers insight into themselves and their students as learners! It serves as a tool to navigate mathematical struggle and understanding with authentic examples and highly applicable techniques for setting the stage, responding during the learning cycle, and reflecting after learning. *Productive Math Struggle* allows for our beliefs about quality mathematics instruction to be transformed into classroom actions that will ultimately serve our students in ways that will positively impact their mathematical development and success."

Jaime Junker
Principal
USD 259, Harry Street Elementary School
Wichita, KS

"Before today I couldn't put my finger on what productive struggle is and how I can help foster it in my classroom. After reading *Productive Math Struggle*, I have a firm grasp on how teachers can use struggle to change how students approach their tasks and the subject as a whole. The book combines a clear case for why we should embrace the struggle, with dozens of practical tools that show teachers, coaches, and administrators can start this work immediately. The authors explain why our students need a culture that can support productive struggle and suggest a way to challenge the prevalent negative beliefs about math that students bring into our classrooms."

Carl Oliver
Assistant Principal
City-As-School High School
New York, NY

"First ask yourself, as SanGiovanni, Katt, and Dykema probe: "Have your students fallen out of love with math?" Or, perhaps, your students were never in love with math because they saw it as inaccessible or confusing. Simply telling students to try harder doesn't work and leaves students and teachers rolling about in a cycle of destructive struggle. Fret no more—the answer is here! When teachers use these fantastic, thoughtful strategic action steps, this heartbreaking phenomenon does not have to be a reality in anyone's classroom! Get this book and begin your journey for cultivating a positive classroom environment that promotes productive struggle that will have your students joyfully falling head over heels in love with mathematics."

Beth Kobett
Associate Professor, School of Education
Stevenson University
NCTM Board of Directors

""Invite productive struggle—don't scaffold it away." This might be my new motto for teaching and learning. This book is an essential read for everyone interested in helping students understand mathematics, not just learn procedures to pass a test. By sharing practical strategies and structures, including focus on mathematical identity and families, the authors help us invite productive struggle into our classrooms. They ask us to think critically about common practices like modifying tasks or revoicing to ensure our use of these strategies supports students for deep learning. Every educator should read and use this book to welcome productive struggle into the classroom."

Sara Delano Moore
Author of *Visible Learning for Mathematics* and *Mathematize It!*
Director of Professional Learning, ORIGO Education

PRODUCTIVE MATH STRUGGLE

PRODUCTIVE MATH STRUGGLE

A 6-Point Action Plan for Fostering Perseverance

John J. SanGiovanni, Susie Katt, and Kevin J. Dykema

Foreword by Matt Larson

CORWIN Mathematics

CORWIN
A SAGE Publishing Company

For information:

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A SAGE Company
2455 Teller Road
Thousand Oaks, California 91320
(800) 233–9936
www.corwin.com

SAGE Publications Ltd.
1 Oliver's Yard
55 City Road
London, EC1Y 1SP
United Kingdom

SAGE Publications India Pvt. Ltd.
B 1/I 1 Mohan Cooperative Industrial Area
Mathura Road, New Delhi 110 044
India

SAGE Publications Asia-Pacific Pte. Ltd.
18 Cross Street #10–10/11/12
China Square Central
Singapore 048423

Printed in the United States of America.

ISBN 978-1-5443-6946-4

Executive Editor, Mathematics: Erin Null
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Copy Editor: Terri Lee Paulsen
Typesetter: Integra
Proofreader: Scott Oney
Indexer: Integra
Cover Designer: Gail Buschman
Marketing Manager: Margaret O'Connor

This book is printed on acid-free paper.

20 21 22 23 24 10 9 8 7 6 5 4 3 2 1

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FOREWORD

Seldom has a book been as timely or as necessary as *Productive Math Struggle* is today. The cumulative evidence is clear that when students engage in productive math struggle it can lead to a deeper understanding of mathematics, the development of a positive mathematical identity, and the continued study of mathematics. Productive math struggle has become a major theme at education conferences, institutes, and workshops all across the country. The concept of productive math struggle is so omnipresent that it is one of those rare educational concepts to make its way into mainstream media and popular culture.

While productive math struggle might be widely discussed, it is also the case in the United States that far too many parents, students, and even teachers simply don't believe there should be any struggle, productive or otherwise, in the mathematics classroom. The prevailing societal expectation is the myth that an effective teacher of mathematics makes the mathematics easy for students. In turn this leads to the unproductive belief that if a student experiences struggle in mathematics, then either the teacher is not doing his or her job well, or the student simply "isn't a math person." Ironically, many of the individuals who hold these negative beliefs about productive math struggle are simultaneously strong believers in productive struggle if the construct is applied to music or athletics. In performance activities like music or athletics, most parents, learners, teachers, and coaches expect participants to experience challenge and struggle, make mistakes, and continue to learn, grow, and improve. Productive math struggle implies that the process of learning mathematics is not fundamentally different than the process of mastering a piece on the piano or improving one's time in the one-mile run.

While seemingly everyone is talking about productive struggle, no one is providing the comprehensive guidance necessary for teachers and leaders to effectively nurture and support student engagement with productive math struggle. A clear gap exists between what we know about the importance of productive math struggle and direction on how to successfully engage students in productive math struggle. Specific guidance on how to overcome this gap is what practitioners desperately need and what makes *Productive Math Struggle* an indispensable resource for every teacher of mathematics. Grounded in the research as well as their personal experiences as teachers, SanGiovanni, Katt, and Dykema provide six action steps teachers can leverage to intentionally plan to provoke and support productive math struggle before, during, and after a lesson. Math struggle is typically viewed as an unfortunate and unexpected

event that emerges during a lesson, but the authors successfully reframe it as something teachers of mathematics need to intentionally plan for students to experience.

The authors illustrate with vivid examples how many of our "taken for granted" instructional practices and structures can undermine productive math struggle and damage students' mathematical identity. In addition to the action steps, the authors provide numerous supporting routines, activities, teaching tips, vignettes for reflection, and "struggle moves" teachers can implement to nurture an environment where students successfully engage in productive math struggle and "push through" that struggle to gain a deeper understanding of mathematics.

Coherence in education is at best elusive. One of the remarkable accomplishments of SanGiovanni, Katt, and Dykema's work lies in how they seamlessly connect the research on high-quality tasks, high expectations, identity, and *equity* to productive math struggle. This is perhaps their greatest contribution. The authors see productive math struggle as a critical feature of mathematics classrooms that supports access, equity, and empowerment, specifically arguing that every student is "worthy of struggle." Productive math struggle isn't just for students in gifted, talented, or AP courses. Productive math struggle is for each and every student. SanGiovanni, Katt, and Dykema make a compelling moral argument that productive math struggle is something to be valued, rather than something to be avoided—equity demands that each and every student experience productive math struggle.

Mathematics education in the United States finds itself, as it so often does, at a crossroads between what the research indicates makes a positive difference for teaching and learning on the one hand, and dominant societal beliefs concerning mathematics and mathematics instruction on the other—beliefs that in the case of productive math struggle stand as obstacles to the implementation of the very practices needed to reach every student and achieve more equitable outcomes. SanGiovanni, Katt, and Dykema provide a clear pathway through these obstacles. In an era where "helicopter" and "bulldozer" parents rush to rescue and eliminate all struggle from their children's lives, productive math struggle becomes something much more than an instructional experience; it becomes a necessary transferrable life skill.

I encourage you to collaboratively engage with your colleagues to study, implement, and reflect on the pathway the authors offer. Demonstrate to yourself and your colleagues that you practice productive struggle by continuing to learn, reflect, and grow in your practice of teaching mathematics. By doing so you will not only deepen your students' understanding of mathematics, but you may ultimately prepare your students to better navigate life itself.

Matt Larson, PhD
Past President (2016–2018), National Council of Teachers of Mathematics (NCTM)
Associate Superintendent for Instruction, Lincoln Public Schools, NE

ACKNOWLEDGMENTS

This book has been a collaborative effort. We are grateful to Corwin for making it a reality. We are thankful that they recognize the importance of productive struggle in the mathematics classroom. We appreciate that they, too, know that each and every student is capable of productive struggle when afforded the opportunities to do so.

Susie Katt would like to thank John and Kevin for being an amazing team to work with! She thanks her Nebraska colleagues, who inspire her with their passion and strong commitment to teaching and learning mathematics. She thanks Matt Larson for his wisdom and guidance, and her closest friends for their constant encouragement. Susie thanks her parents, as they taught her what hard work can accomplish. And most importantly, she would like to send a heartfelt thank-you to her family, Jason, Tenley, and Huxton, for their patience, support, and never-ending love, which made it possible for her dream of authoring a book to come true.

Kevin Dykema would like to thank his wife, Mary, and two daughters, Abigail and Lindsey, for their encouragement and support through this project and for suggesting for the past few years that he write a book. Kevin would also like to thank his parents, Marv and Jan, for their constant love and support. Kevin also thanks John and Susie for being a great team and allowing him this wonderful opportunity. Finally, Kevin would like to thank all his past and current students who have taught him so much about the need to support productive struggle in learning math.

John SanGiovanni thanks his family—especially Kristen—who has taught him more about struggle, perseverance, and happiness than he could have ever imagined. He thanks his parents for giving him a foundation for success. He thanks his many math friends and coworkers for making him better. He is always grateful to those special math friends that gave him opportunities to learn and grow professionally. And, of course, a special thank-you to Susie and Kevin for taking on this project, being partners throughout, and doing so with responsiveness, grace, and humor.

Lastly, thanks to the staff at Corwin for transforming a featureless document file into such an appealing, practical tool for teaching and learning. Thank you to Paula Stacey for her work and questions to make this project the best it can be. Special thanks to Erin Null (again) for her enthusiasm, partnership, thoughtful questions, insight, and friendship.

Publisher's Acknowledgments

Corwin gratefully acknowledges the contributions of the following reviewers:

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Jennifer Bay-Williams
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Pewee Valley, KY

Melissa J. Black
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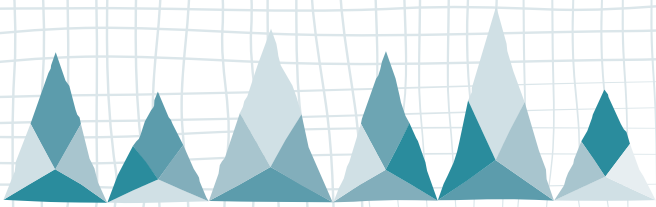
INTRODUCTION
WHY STRUGGLE? WHY NOW?



Each of us, as teachers and as students, have experienced struggle in some way. Still, it's only recently that new thinking about the role of mindset and grit have helped us truly "find" the value of struggle. That movement has been invaluable in setting the stage for us to nurture consistent, *productive* struggle and perseverance on the part of students in mathematics classrooms. In spite of this progress, direct actions to provoke, support, and develop students' comfort with struggle have been lacking or loosely connected. This lack prompted the three of us to think about how we learned to facilitate and channel students' struggle in productive ways.

We wrote this book to help colleagues realize "productive struggle success." We wanted to provide specific strategies and routines to put into action and a coherent approach for doing so. But before we could get started, we had to make better sense of who we were and who we are now relative to struggle. We had to think about our journey with struggle as students, as teachers, and as professional developers.

The Struggle Experiences as Students and Teachers

The three of us (John, Susie, and Kevin) grew up in different parts of the United States. We went to different schools. We are different ages. Yet, each of us remembers that learning math meant acquiring steps and procedures for doing math. Although we didn't realize it at the time, what we lacked back then was a deep understanding of the concepts behind the steps. As the saying goes, ours was not to ask why. We could *do* math but that isn't to say that we *knew* math.

We were led to believe that there was a right way to do math. We were told that we would get the answer if we just followed "these steps." We developed notions that certain people were good at math and those people were almost always faster than others. We were led to believe that struggle was bad. Good math students didn't struggle.

But we did struggle from time to time. We struggled with counting coins and making change to computing and comparing fractions through mechanical manipulation of common denominators. Relying on the "FOIL method" for multiplying binomials and factoring polynomials didn't help us understand what we were doing or why. Our "understanding" was fleeting and fragile. It faded quickly. This loss of understanding undermined our confidence.

We aspired to teach and trained to be teachers. We landed in primary, intermediate, and middle school classes where our primary challenges were to find clever ways to engage students, figure out how to make math clear, and know what to do when our students didn't get it.

All three of us remember getting accolades as student teachers for being able to explain math in different ways when our students didn't get it the first time. We carried those to our first posts. And we soon became frustrated with our students because it felt as though we were doing all of the work. They had become our mathematics dependents. They waited to be told what to do. They said "I don't know," and we jumped to rescue them.

We were also uneasy about district and state tests. Lacking the confidence that our students retained what they seemed to know only a few weeks before, we felt the need, if not the requirement, to cram and review, knowing that we would be judged by others based on those results, just as we judged ourselves as well on a daily basis. Like most teachers, we were satisfied when our students got it. We were frustrated and dejected on days when they didn't. In many ways our early experiences of teaching math were strikingly similar to the experiences we had as students. Our focus, at its core, was still mastering procedures and steps.

What We've Learned About Teaching Math and Struggle

As we pursued opportunities to learn and grow our craft, attending numerous professional learning sessions, reading countless books and articles about teaching math, and attending graduate school, exchanging ideas with colleagues and mentors, our awareness of our own struggles and our own learning grew. We noticed our professors valued discussion, insight, and making sense over leading us directly through steps and solutions. We were reminded of the satisfaction of overcoming challenges in learning settings. These experiences strengthened our value of struggle, recognizing it as meaningful, worthwhile, and productive.

Our work with teachers, coaches, principals, parents, and students across the country has only deepened and enhanced our understanding of struggle. We have learned that all teachers are challenged with understanding the very notion of productive struggle. How can struggle be productive? Is struggle a sign of failure? Of poor teaching? But maybe the biggest issue teachers face with struggle is their own desire to see their students succeed. Caring so very much about their students, teachers become uneasy when they see their students struggling, wondering if it will erode students' confidence in their own capabilities. Lacking confidence in the value of such struggle, teachers resort to what they know, focusing on what they, and we all have, assumed are tried-and-true approaches: memorization, step-by-step instructions, and guiding students down the path to the correct answers, rather than guiding their thinking so they can find their own path. We learned that well-intended teacher actions can compromise or even completely avoid struggle in order to achieve what might on the surface look like success.

This avoidance can have profound consequences for equity and access. Depriving some students of experiencing struggle robs them of valuable learning opportunities. *All students deserve the right to struggle*. It cannot be reserved for the students who appear to need extra challenge or enrichment. Conversely, those students who need extensive reteaching may be left to engage in a constant state of *unproductive* struggle. And young students, just because they are new to formal learning, cannot be sheltered from struggle. Each and every student must have access to high-level mathematics—opportunities to think, discuss, and learn how to struggle productively. It begins with student identity, agency, and the belief that one can.

Depriving some students of experiencing struggle robs them of valuable learning opportunities. All students deserve the right to struggle. It cannot be reserved for the students who appear to need extra challenge or enrichment.

The Current Challenge

Beliefs that struggle can be productive have gained momentum in recent years, and notions about growth mindset and making mistakes are alive and well. These are important first steps. Yet, proclamation is not realization. Words and actions have not intersected as much as we would like.

In many classes today, students express awareness that effort matters and acknowledge the importance of taking chances and accepting mistakes. But while their attitudes relative to mistakes and effort are changing, their actions and reactions have not yet caught up. Teachers are challenged to provide effective responses that help students engage in struggle and push through it. Teachers need help with how to take action. Their need comes as no surprise.

Teacher preparatory programs do many things well. Preparation tends to focus heavily on areas of teaching as lesson planning and improving instructional resources. However, supporting students through differentiation and permanent scaffolding in order to *avoid* struggle seems to take precedence over responding to and navigating struggle. On top of this, many programs require only a single course in mathematics pedagogy and content. These limitations hamper preservice teachers' opportunity to learn about planning for, supporting, and even embracing struggle. Simply put, new teachers are not readily prepared to handle the situations of struggle in their own classrooms.

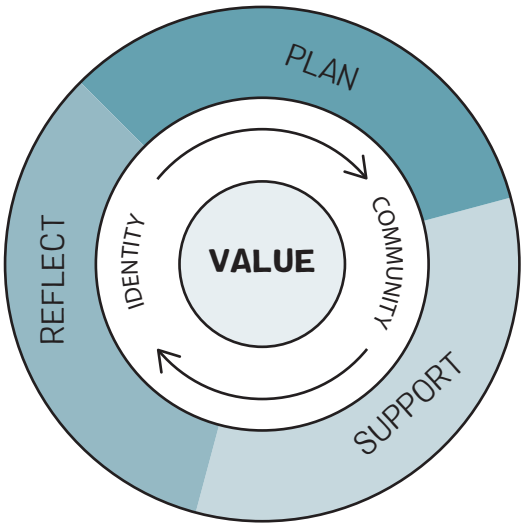
Once on the job, teachers can find that professional learning about productive struggle to be lacking. Administrators, curriculum offices, and district initiatives compete for valuable professional learning minutes. There is only so much time allotted to develop mathematics content and pedagogy. And even though there is increasing recognition of the value of productive struggle in the learning sessions that are offered, teachers may find that more cheerleading and messaging in support of struggle are delivered than actual specific insights, actions, and strategies for navigating it and even helping it thrive in the classroom.

There are many misconceptions about promoting struggle in the classroom. One is that struggle is something that happens "in the moment," seemingly spontaneously and unpredictably *during* the lesson and so is impossible to plan and prepare for. But we are here to tell you that struggle is not something that happens by chance. Indeed, we have learned that by making purposeful decisions before, during, and after the lesson, teachers can teach students, all students, to engage in productive struggle. There are distinct actions within those three phases. This book outlines six essential actions for nurturing productive

struggle *before* (Actions 1, 2, 3, and 4), *during* (Action 5), and *after* the lesson (Action 6). The relationship between these actions might be thought of as shown in Figure i.1.

- Action 1: Value Productive Struggle
- Action 2: Foster Identity
- Action 3: Build Community for Productive Struggle
- Action 4: Plan for Productive Struggle
- Action 5: Support Productive Struggle
- Action 6: Reflect on Productive Struggle

Figure i.1 • Productive Struggle Actions



How This Book Can Help

"The struggle is real" is a popular catchphrase in education. But all too often it is just that, a saying that isn't accompanied with what actions a teacher can take. This book intends to help you take action to support productive student struggle by describing what these six actions entail and how you can bring them to life in your work through the collection of specific activities. It provides opportunities for you to reflect on where you are with productive struggle and help you consider what aspects of your professional practice work well and where you would like to grow.

It is important to keep in mind that this book isn't a recipe for success. Navigating productive struggle is hard and complex. It takes time. Student struggle is not something that is "fixed" by the teacher. It is a process that is facilitated by the teacher. While there is no magic bullet to produce

Student struggle is not something that is "fixed" by the teacher. It is a process that is facilitated by the teacher.

productive struggle, we do believe that there is a game plan for it. These actions and activities can help you develop your game plan for struggle.

The many vignettes in this book are based on our own experiences as teachers and experiences of others we have observed and talked to. They are meant to illustrate concepts offered in each chapter and the practices that may to varying degrees support or not support them. To be clear, we *all* have been to some extent each teacher in each vignette. Neither teacher is good or bad, better or worse. They simply reveal different approaches and perspectives. Some may promote productive struggle while others might undermine, restrict, or avoid it. The vignettes are intended to help you, as they have helped us, think about what we want to do to enhance our practice.

Three Things Before You Begin

Before you dig in, take a moment to reflect on how you feel about the idea of struggle by addressing the question below and completing the inventory that follows. We'll ask you to come back to these thoughts in different parts of the book. At the end of the book, we'll invite you to revisit the reflection, definition, and inventory that you complete here. It will be an opportunity for you to get a sense of how this work has helped you grow professionally. Perhaps your struggle with struggle is just beginning. Maybe you are using this book as an opportunity to revisit or reinforce or enhance approaches you are already trying out. Wherever you are on this professional journey, we invite you to join us in the struggle.

Reflection Question:

What is a struggle experience you have had as a student and one you have had as a teacher? What effects have each of these experiences had on you?

Definition:

How do you define productive struggle?

Figure i.2 • Productive Struggle Self-Inventory

Productive Struggle Self-Inventory

Where are you with productive struggle? Take a moment to rate yourself on the teacher survey below. Rate each question from 1 to 10, with 1 being never and 10 being always.

I am aware of my math identity.

1 2 3 4 5 6 7 8 9 10

Never Always

I am aware of my students' math identities.

1 2 3 4 5 6 7 8 9 10

Never Always

I actively and consistently reinforce math community.

1 2 3 4 5 6 7 8 9 10

Never Always

I select high-quality mathematics tasks.

1 2 3 4 5 6 7 8 9 10

Never Always

I modify tasks to provoke struggle.

1 2 3 4 5 6 7 8 9 10

Never Always

I modify tasks to make them easier for students to work with.

1 2 3 4 5 6 7 8 9 10

Never Always

I complete the math task before I use it with my students.

1 2 3 4 5 6 7 8 9 10

Never Always

I anticipate what students will do and think about questions to ask them in response.

1 2 3 4 5 6 7 8 9 10

Never Always

I pause in the middle of group work to discuss progress and challenges.

1 2 3 4 5 6 7 8 9 10

Never Always

My students ask questions or refer to resources when they are struggling.

1 2 3 4 5 6 7 8 9 10

Never Always

We reflect on struggles and strategies for overcoming them during class discussions.

1 2 3 4 5 6 7 8 9 10

Never Always

I reflect on my students' struggles and the strategies they use to overcome them.

1 2 3 4 5 6 7 8 9 10

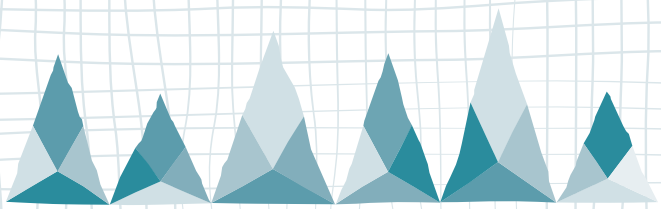
Never Always

This self-inventory can be downloaded at
resources.corwin.com/productivemathstruggle

CHAPTER 1

ACTION 1

VALUE PRODUCTIVE STRUGGLE



At a recent middle school mathematics team meeting, Mr. Jones and Mrs. Sanchez shared with each other how they taught proportions to their students, a topic they covered several weeks ago. Both acknowledged that this can be a frustrating topic for many of their students. They both had selected the same initial problem to pose to students.

> *"If four shirts cost $112, how much will 10 shirts cost? Assume all the shirts are the same price."*

All tasks in this chapter can be downloaded from resources.corwin.com/productivemathstruggle

Mr. Jones started the conversation, observing that he spent a great deal of time thinking about what has made this difficult for his students in the past. His solution was to create a slide to display with notes that he thought would clearly explain how to efficiently solve proportions utilizing "cross multiply and divide." He had students copy the notes down and use different colors to highlight where to place the numbers to

Chapter icon: shutterstock/topform

follow the cross multiply and divide algorithm. To keep students actively engaged, Mr. Jones asked frequent questions during the note taking. After the notes were copied, he gave them some practice problems. Most students referred back to the step-by-step directions in their notes to successfully complete those problems.

Mr. Jones concluded the class by giving students a similar problem as an exit ticket. On looking them over after school and noting that the students were very successful, he felt good that he had removed much of the frustration for the students with his clear notes. Unfortunately, that good feeling did not last. At a team meeting the following month, he expressed frustration that many students had forgotten in the past couple of weeks how to solve proportions and that some had lost their notes. He felt that he was constantly reteaching with more than a few of his students.

Mrs. Sanchez reported that she took an entirely different approach. She decided against providing notes with step-by-step directions, which she felt led students to memorize what to do to get an answer rather than to understand the concept. Instead, she started the lesson by letting the students know that they would be learning to solve a new type of problem, that it might initially be a bit frustrating, but that she was confident they would figure it out. She reminded them that it's normal to be confused and even unsuccessful with something new. She then posed the shirts problem.

To avoid monitoring student work too soon, and as a way to signal to students that she was not yet available to answer questions and provide help, she went to her desk and pretended to be absorbed in something else. Her goal was to have students rely on their group mates instead of her.

When she left her desk a few minutes later to circulate and monitor progress, she asked groups to explain what they were doing. She learned that groups were using a variety of methods to solve the problem. For example, one group found that each shirt cost $28 and then multiplied by 10 shirts to get $280. Another group realized that they want 2.5 times as many shirts (10 ÷ 4 = 2.5) and then multiplied 112 by 2.5 to get $280. A third group halved the number of shirts and price. They figured that two shirts would cost $56, having calculated that 10 is 5 times 2 so 5 times $56 (280) would be the price of 10 shirts.

Mrs. Sanchez then facilitated a class discussion where groups described the methods they used. She noticed excitement among the students as they connected their strategy to other groups'. She then gave students a couple of problems to do that evening for practice. She told them they could use any strategy that they preferred.

When she met again with her middle school team the following month, she reported that most of her students could still efficiently solve proportion problems. She also noted that she wasn't spending much time reviewing, and a full reteaching wasn't needed at all.

This chapter

- defines what productive struggle is and what it looks like while differentiating it from unproductive and destructive struggle,
- establishes why productive struggle matters relative to the purpose of teaching and learning mathematics as well as the ideals of equity and access,
- makes a case for what classrooms and schools can look like and the kind of learning that can take place when productive struggle is alive and well,
- shares insights into traditional beliefs about mathematics and struggle and how they damage opportunity for struggle, and
- provides strategies and activities to recognize and advance the value of productive struggle.

As teachers, we may think that removing the struggle helps our students. But does it? Think of a time you learned how to do something, maybe learning to dance, playing an instrument, or learning a sport. Think back to when you learned how to drive (or taught your child to drive!). You probably weren't too successful the first time. There was an interesting, worthwhile challenge that lay before you. You knew you *could* figure it out. You knew you *had to* figure it out. You knew it might not be *easy* to figure out and that it might take a lot of practice to build your reflexes and your muscle memory. Learning mathematics is no different. Productive struggle leads to learning.

But supporting this kind of struggle takes practice. And sometimes it's difficult to know where to begin. This chapter describes where it begins. It may not look like an action on the surface, but taking the time to understand and learn what to *value* in productive struggle is the first step in helping students actually *engage* productively in struggle.

Why Struggle Matters

The case for productive struggle in mathematics teaching and learning is strong. We know that instruction that features productive struggle provides opportunities to delve more deeply into the meaning of math (National Council of Teachers of Mathematics [NCTM], 2014). Productive struggle is necessary for learning mathematics with understanding (Hiebert & Grouws, 2007). It moves instruction beyond a focus on recall and rote procedure. It redefines student and teacher success by extending the meaning of math beyond finding answers. Making use of productive struggle leads to long-term benefits (Kapur, 2010). Students who struggle and overcome that struggle not only develop their own understanding of the math content but also develop new skills and strategies for overcoming new challenges inside and outside of math class. Struggle develops perseverance, resilience, and grit.

Productive struggle can be compromised by a number of factors, such as when students and teachers approach mathematics with fixed mindsets about who can and can't succeed. Carol Dweck notes that fixed mindsets tend to be more prevalent in mathematics (Dweck, 2006), which isn't surprising as the notion that students should be encouraged and taught to productively struggle has not traditionally been a hallmark of mathematics instruction. Struggle in mathematics is less apparent in U.S. classrooms than in other high-achieving countries (Banilower, Boyd, Pasley, & Weiss, 2006; Hiebert & Stigler, 2004). It's also not surprising that many of those countries are perceived to be better at teaching, learning, and doing math than their American counterparts who value productive struggle less.

Students who struggle and overcome that struggle not only develop their own understanding of the math content but also develop new skills and strategies for overcoming new challenges inside and outside of math class.

American attitudes about the value of struggle have contaminated how math and success in math is defined, but that is changing. In its landmark publication, *Principles to Actions: Ensuring Success for All* (NCTM, 2014), the National Council of Teachers of Mathematics (NCTM) included *supporting productive struggle* as one of eight research-informed highly effective mathematics teaching practices. See the list below.

Mathematics Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- **Support productive struggle in learning mathematics.**
- Elicit and use evidence of student thinking.

In that work, NCTM declared that "effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (NCTM, 2014, p. 21).

Struggle lies at the core of all that math can be and should be. We argue that it matters for several reasons. Experiencing productive struggle establishes the following.

- Math is more than the pursuit of answers. Math is a way of thinking and understanding.
- Math isn't an acquired procedure. Students are not "sheep" who blindly do math the way they are told to do it and/or simply see it as a set of procedures; they are competent people who are able to make their own sense of math.
- Math is for each and every one of us. Productive struggle is about equity, access, and opportunity. We need to ensure that each and every student must have access and opportunity to productively struggle.
- Struggle, in general, is essential for learning and living.

Math Is More Than the Pursuit of Answers

Many of you may have experienced mathematics instruction that could be described as the "pursuit of answers," instruction that emphasized steps and procedures for computing and finding unknowns. Even word problems and applications had underlying procedures designed to ensure that correct solutions were found. Key word strategies as well as "circling numbers" and "highlighting questions" are just two examples. Discussion likely focused on determining whether procedures were carried out accurately. Discussion about efficiency, representation, estimation, relationship, and variation of strategy was rare. The message was that the purpose of math class was to acquire someone else's approach to "doing" mathematics. The focus was more on *doing* math rather than on *knowing* math.

What is math to you?

Viewing mathematics as a pursuit of answers changes the focus of the struggle from understanding and empowerment to replicating someone else's approach to doing math, committing disconnected steps to memory, and doing so quickly. For many, that struggle became destructive as the purpose of math class was mostly about *doing* math rather than to *knowing and using* math. Productive struggle cannot be valued if math is not valued beyond right answers.

Math Isn't a Procedure

Perceiving math to be about answers and "correct strategies," often offered in an attempt to avoid or bypass student struggle, can cause mathematics instruction to be overly, if not completely, proceduralized. This approach to math takes many forms. It appears as "rules" for filling 10 frames. It is the hunting and highlighting of specific key words in word problems. It

is reliance on chants such as "five and above, give it a shove" for rounding. It allows "tricks" and mnemonics like KFC ("keep, flip, change") often used in division with fractions, to become substitutes for the real language and concepts (e.g., "I'm going to KFC it."). It requires students to use the "FOIL method" because they don't understand the distributive property.

Productive struggle cannot be valued if math is not valued beyond right answers.

Such approaches to instruction risk turning students into "math sheep," who rely on steps, rules, permissions, and "tricks" to do math. Any struggle they engage in is focused on memorization of and adherence to someone else's approach, not on thinking and reasoning or engaging in the struggle necessary to spur growth in thought and reason. These approaches also signal that rules, procedures, and solutions alone are the values of math and that struggle is unworthy or unnecessary.

Consider this:

Students in two different first-grade classrooms have been working with different problem-solving situations. Recently, they were working with "take-apart" problems with both addends unknown. The task they were working with was:

> Frankie has nine fish and two fishbowls. What are different ways Frankie can put her fish into the two fishbowls?

As students begin working with partners on this task, it becomes evident the students are struggling to get started. What unfolds next is different in the two classrooms.

Classroom A:

Ms. Anderson asks the students to meet her back at the rug. She tells students they need to use manipulatives. "If you use cubes, you can use these paper plates as your bowls. If you put one fish in this bowl, the other eight fish go in this one. Then you can record it on your paper, like this: 1 + 8. After that, you just have to move one fish from this bowl to the other bowl. You can keep doing this to find all of the ways to put the nine fish into the two fishbowls."

The students return to their respective work areas, grabbing cubes and paper plates on the way. They arrange the plates and cubes on their table as Ms. Anderson had done. Students methodically move one cube from the plate of eight to the other, and record the equations on their papers. Ms. Anderson monitors the room and selects one pair

of students to model how they found all of the combinations on the board. She ends the task by saying, "Good job. It will be important for you to remember this for tomorrow because we will be working with a very similar problem."

Classroom B:

Ms. Jaye directs students to try and solve the problem. She poses questions to the students as she walks around the room. When she notices pairs of students struggling to get started, she asks them what they know about the problem. She rephrases what the student says: "You said you know Frankie has two fishbowls and nine fish. You said you need to find different ways Frankie could put her fish into the two bowls. So, what might you try to get started?" Ms. Jaye listens to the student response, "I am going to draw a picture of two fishbowls," and then says, "That would be a great way to begin. I'll check back with you in a few minutes to see how you are doing." Ms. Jaye walks away from the pair of students, leaving them to get started on their own. She moves around the room, checking in with other pairs of students. As she observes the different approaches students have started using on their own, she is thinking about which student strategies she wants to share during the whole-class discussion about the problem.

Both Ms. Anderson and Ms. Jaye feel as if they had orchestrated a successful lesson. Students were working on the task, engaged in the math content, coming up with correct answers, and working well in groups. However, there were stark differences in how the two teachers approached the task. Ms. Anderson limited any opportunity for struggle, if not avoided it altogether, by outlining a series of steps for her students to follow to ensure all of her students arrived at the correct answers. If there was struggle, it was not about the meaning of the math. Ms. Jaye asked questions to prompt student thinking during struggle, as her goal was to engage the students in thinking so they could stick with the task to arrive at an answer.

The instructional moves of both teachers were intended to provide assistance for students as they were struggling with the task. However, one teacher removed the struggle by proceduralizing the mathematics. By outlining the steps to complete the task, Ms. Anderson increased the likelihood of students successfully completing it and finding the correct answer. Whether students understood the problem or the math involved in taking apart "nine" remained unclear.

In contrast, by asking questions to facilitate student thinking and working to equip students with strategies they could draw upon again, Ms. Jaye ensured that her students would be better prepared for the next

time they encounter struggle. More important, they likely came away from the task with a better sense of decomposing nine, take-apart problems, and subtraction. Their understanding can be generalized and transferred to other numbers as well.

Math Is About Equity, Access, and Opportunity

Of the classrooms profiled so far in this chapter, which would you rather attend? Which classroom would you rather your son or daughter be a part of? Why? There are clear differences between the two classrooms in each vignette, with each presenting different opportunities for thinking, doing, understanding concepts, applying, and transferring understanding to new situations or problems.

Students in Mrs. Sanchez's sixth-grade classroom and those in Ms. Jaye's first-grade classroom have access to opportunities to struggle. Those students have opportunities to make their own sense of mathematics. They experience struggle and come in contact with strategies for overcoming it. But not all students are given such opportunities, and these incongruent experiences of math instruction pose a considerable threat to equity and access. Such incongruities don't necessarily exist because of policies or curricula in different states or school districts. As these examples illustrate, the struggle inequity can just as easily occur in classrooms that are in the same building.

NCTM identifies commitment to equity and access as an essential, non-negotiable element of successful mathematics instruction. The council states that "an excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (NCTM, 2014, p. 59). Access and opportunity to struggle are included in this powerful proclamation. It directly relates to the curriculum and instructional resources that provoke struggle. It connects with pedagogy that supports productive struggle. It stands for high expectations necessary for engaging in struggle.

There are moments when explicit instruction of a skill or concept must be provided. But before such instruction happens, students must have the first crack at making their own meaning. This means resisting the watering down of rich tasks and recognizing diverse approaches to thinking about and carrying out mathematics. It means we avoid re-creating *our own* understanding of math in our students.

High expectations for each and every student are prerequisites for productive struggle. It is rooted in valuing struggle and believing that each

and every child is not only capable of struggle but also worthy of struggle. Choosing to provoke struggle or avoid it creates opportunity gaps for students who lose out on the chance to develop problem-solving skills and reasoning. Each and every student is capable of doing math on their own and for their own reasons. By challenging them and supporting them through struggle, you can help them build the strategies and thinking they can carry with them into their futures. You empower them to see that they are in control of their learning rather than reliant on someone for their learning. Ensuring equitable access to struggle means embracing its value and doing the hard work to make sure the right kind of struggle is pursued and nurtured in our schools.

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Productive Struggle Is Essential for Living and Learning

When students experience struggle in mathematics, they gain more than learning just about the subject. They begin to realize that productive struggle is an essential part of learning, no matter the content. In addition, they have opportunities to grow and refine 21st century skills that will contribute to their overall success in school and beyond (Thoughtful Learning, 2019). When working through struggle, students learn how to think critically and creatively. They develop communication and collaboration skills. They learn to persevere and not give up.

Struggle associated with learning transcends school into the daily lives of students. As students enter the workforce, most will experience struggle during the first few days or weeks of a new job. Many will struggle when they learn to care for a child or begin to manage a home. It is almost a guarantee that each and every student will experience struggle during multiple points in their lives. The lessons students learn, and the skills they acquire by successfully working through struggle in school, will set them up to be more successful in situations when they face struggle in their future.

What Productive Struggle Is and Isn't

We must know what productive struggle is to value it, plan for it, look for it, and support it. To define productive struggle, let's first consider each word separately. *Productive* tells us that something is worthwhile, that it makes progress, or achieves a goal. But what about *struggle*? On first glance, struggle might seem to communicate something entirely

different, the opposite of progress—images of people thwarted, stuck, foundering, or confused. And it is true struggle is about all of these things. People struggle when they need to do something, get somewhere, but with no clear indication of what exactly to do next. It is, however, this lack of direction that also creates an opportunity for learning and here is where the productive part comes in. Productive struggle can be thought of as *purposefully reacting to an unclear challenge so that progress is made or learning advanced*. When struggle is productive, according to Jackson and Lambert (2010), it can empower students, helping them develop positive attitudes about learning and confidence in their own efficacy as learners.

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Jackson and Lambert (2010) characterize productive struggle as leading to positive dispositions about learning, empowering students with a sense of hope or efficacy, and ultimately yielding results. But, of course, not all struggle is productive. Struggle becomes unproductive when students make no sense of a problem or no progress toward the mathematics goal (Warshauer, 2011). Unproductive struggle can cause teachers and students to want to throw up their hands, asking "What did we just do?," or worse yet, "Why did we just do that?" But beyond being frustrating, unproductive struggle also has the potential to be downright damaging. As Jackson and Lambert (2010) point out, when students consistently struggle and produce nothing, they get the message that learning is not possible, they are inadequate, and they may even begin to question their own personal worthiness.

It can cause someone to fall out of love with math.

Students, educators, and parents come to the idea of struggle with wide-ranging beliefs that can complicate how struggle unfolds in the classroom. Some people believe that a student can't succeed until he or she struggles. Others believe that struggle (unproductive or destructive) is how one "earns their stripes" or that everyone has to deal with struggle or that one should just get over it. These "positions" deflect attention from the real challenge, that is, how do we craft productive struggle and support students as they engage in it?"

What Struggle Looks Like

Knowing what struggle is and what it looks like are two different things. We need to be mindful of how we recognize and interpret the struggle in

How do you know if a student is struggling productively or unproductively?

our math classrooms as it manifests in observable behaviors. Some student behaviors indicate that struggle is productive. Other behaviors may signal the opposite, as students try to avoid the struggle by diverting attention to something else. Here is where struggle can become unproductive and possibly destructive.

Unproductive struggle takes many familiar forms. For some students it looks the same each time. For others, it changes depending on a variety of internal and external factors. Obvious signs of struggle are frustration, anger, and possibly physical altercations. For many students, their reactions are the result of being unable to process or cope with the built-up frustration. Others, seeking to avoid the thing that triggers their struggle, might make a bathroom request or make up a reason to visit the nurse. For some students, struggle overwhelms their emotions and so they cry or just give up and put their heads down. And for some students they distract or deflect the struggle by changing the unwanted attention to mischief or joking behavior.

Conversely, there are a number of different behaviors that communicate a student's willingness to embrace struggle. A student who values struggle may draw a representation in their math notebook. They may get out of their seat to reference a chart that is hanging on the other side of the classroom. Or they may use crayons from their desks as a makeshift manipulative to model a mathematical situation while solving a problem.

These behaviors, when viewed through the lens of misbehavior rather than struggle, can be misinterpreted. For example, a student's turning around to talk with a classmate could be viewed as avoidant behavior. But what if the student is asking their classmate a question? That is a good way to make sense of a problem. Or a student may be drawing on their paper, which resembles doodling from afar; however, the drawing turns out to be a number line that the student is using to solve a multiplication problem. Teacher assumptions about behavior can damage attempts to persevere through struggle.

You also have to consider patterns within the behaviors you observe. Does Hector immediately ask a question every time you tell students to get started? Does Robert always consult with the same group when he has a question? Does Brooke always need to go to the bathroom just before the class is about to play a fact game? You need to ask yourself if these patterns of observed behavior tell you something in relation to struggle. In many instances they will.

In Figure 1.1, we identify a few behaviors that might be evident in a math classroom. The behaviors aren't definitive, and the list is certainly not comprehensive. However, the table may help you reflect upon why a behavior you observe in your own classroom might be happening. If you recognize a number of unproductive behaviors, you can work on diminishing their frequency as you help students develop a deeper appreciation for struggle and how it contributes to their learning. There are a number of suggestions in the upcoming actions that can help you do this.

Figure 1.1 • Behaviors Associated With Productive and Unproductive Struggle

UNPRODUCTIVE STRUGGLE	**Behaviors That May Signal Struggle AVOIDANCE**	**Behaviors That May Signal Struggle DEFLECTION**	**Behaviors That Could Signal Struggle FRUSTRATION**
	• Asking off-task or unrelated questions • Going to the bathroom, getting a drink, sharpening a pencil, going to the nurse • Playing inappropriately with materials • Putting head down • Copying others' work • Talking with classmates	• Making jokes, silliness, teasing • Asking unrelated questions, sharing unrelated stories • Being rough with materials • Acting out	• Arguing with statements or directions • Acting out • Breaking pencils, being rough with materials • Teasing classmates • Looking at what others are doing • Putting head down • Tearing up, crying
PRODUCTIVE STRUGGLE	**Behaviors That May Signal Struggle to UNDERSTAND the Problem**	**Behaviors That May Signal CONSIDERATION of Possible Strategies to Use**	**Behaviors That May Signal How Students ASSESS Their Process or Strategy**
	• Asking on-task or related questions • Using manipulatives • Drawing a representation of the problem	• Getting up to get manipulatives or tools • Talking with a classmate • Asking a classmate for assistance • Walking around the room	• Walking around the room • Looking at reference materials (journals, charts) • Using calculators or other tools • Talking with classmates

When Schools Value Struggle

Productive struggle needs a nurturing environment to flourish and grow. It is most effective when it transcends single classrooms and is valued throughout the mathematics program or school, with all teachers making their own, individual-but-aligned efforts to support it. In such a school, myths about teaching and learning math, as well as about struggle, would be challenged with facts and confidence. Signs of productive struggle would be on display across classrooms.

How does your school show that it values productive struggle?

What are these signs?

The National Council of Teachers of Mathematics identifies teacher and student behaviors for the effective teaching practices of mathematics (NCTM, 2014). NCTM's offerings of student and teacher behaviors are excellent for initial conversations about what is happening when productive struggle is alive and well in mathematics classrooms. But, as one digs more deeply into productive struggle other behaviors emerge for consideration.

Teacher Behaviors for Productive Struggle

Teacher behaviors are a strong indication of the health and well-being of struggle within the mathematics classroom. Knowing these teacher behaviors is essential as there is a cause-and-effect relationship between teacher behaviors and student behaviors. In other words, even things like struggle don't happen randomly in classrooms. The way students do or do not struggle is profoundly influenced by what teachers do and do not do.

NCTM identifies these behaviors. Teachers

- anticipate what students might struggle with during a lesson or problem,
- give students time to struggle,
- ask questions that scaffold thinking,
- help students realize that learning comes from confusion and errors, and
- praise students for their effort and perseverance (NCTM, 2014).

The behaviors above are a great place to start. But we would add that for productive struggle to be *most* effective teachers must also do the following.

- Know their students as individuals with strengths, needs, interests, and fears.
- Continuously develop and reinforce a positive classroom community.
- Select or modify tasks in order to provoke productive struggle.
- Consider how students will react to struggle as it occurs.
- Take the "collective pulse" of the class during lessons.
- Reinforce what students can do as they begin to struggle.
- Debrief student struggle and effective action with similar emphasis on the steps used for finding a solution.
- Reflect on how students engaged during the lesson and how they (the teacher) reacted to the struggle.

Student Behaviors for Productive Struggle

Student behaviors indicate what is going well relative to struggle (or any teaching strategy) and what needs to be developed. Therefore, knowing student behaviors of productive struggle is as important as knowing teacher behaviors. Looking for these behaviors can help you take stock of the frequency and ongoing development of productive struggle within your students. It can help you reflect on your practice and your mathematics ecosystem. For that reason, NCTM also identifies productive struggle behaviors for students. Productive struggle is in evidence when students

- know that learning comes from struggle,
- ask questions related to the struggle,
- persevere, and
- help one another without giving the answer or solution (NCTM, 2014).

As with teacher behaviors, there are other student behaviors when considering the health of productive struggle in the classroom. We add that students should also do the following.

- Believe they are a valuable member of the community.
- See themselves as doers of mathematics.
- Engage in problems and tasks.
- Show respect for others' ideas, opinions, and possible solutions.
- Know when to ask questions.
- Identify what questions to ask.
- Attempt to rectify struggles on their own prior to asking for help.
- Participate in lesson reflection.
- Recognize that making mistakes is a normal part of learning.

Knowing these behaviors helps. After all, they are not developed in students without awareness of, attention to, or planning for them. For the most part, students do not come to us with the behaviors fully in place. Productive struggle behaviors are not usually inherent but must be taught and nurtured. This means making sure all stakeholders—students, teacher, parents, and school personnel—are aware of what these behaviors are and why they are important.

Educating Families

Many parents and other family members have had negative experiences with mathematics themselves. They may perceive the purpose of instruction to be grounded primarily in showing students how to do a procedure—likely the way they learned themselves. By educating them about the kinds of student behaviors associated with productive struggle and the purpose of these behaviors, teachers and schools can help parents learn to value struggle as well. These behaviors can be communicated in a myriad of ways. They could be a component of the back-to-school message. They could be included in school newsletters, with blurbs about why they matter or how behaviors can be supported at home. Reflections or journal opportunities can be assigned for homework in which students might be asked to discuss the behavior with parents seeking "home examples" or "non-math" examples of the behavior. When communicating with families, you want to address the following important questions about struggle:

- Why does struggle matter?
- How does struggle help?
- How does student identity affect struggle?
- How can they help students foster positive math identity?
- What parts of math do students often struggle with?
- How will instruction support student struggle?
- What can they do at home to help students when they struggle (with anything)?

Productive Struggle Look-Fors

Teachers, principals, and district leaders routinely collect evidence to make decisions about instructional strategies and progress. But, faulty indicators or misguided perceptions can lead to poor adjustments. Understanding what kinds of productive struggle behaviors to look for is important when considering the vitality of productive struggle in the classroom. Below, we present a productive struggle walkthrough tool for teacher behaviors, student behaviors, or both.

As a teacher, you could set a yearly professional goal that takes aim at improving productive struggle. This tool can be used to collect evidence recognizing progress toward the goal. Note that the number of behaviors to "look for" may need to be adjusted based on the amount of time spent in a classroom. To measure growth, you could set goals for increasing the frequency of occurrence of a certain behavior(s) or all of the behaviors as the year progresses. Of course, the tool could also be simply used to collect evidence for personal, periodic reflections.

This tool could be coupled with the Productive Struggle Self-Inventory from the Introduction (page 7). To do this, you can complete the inventory at the beginning of the school year and again at the end of the year, with a goal of increasing certain, if not all, statements on the survey.

Principals or district leaders have a stake in looking for the right signs of student struggle and the teacher behaviors that elicit it. They too can use the productive struggle walkthrough tool. They can use the tool to look for teacher behaviors, student behaviors, or both. Elements of a productive struggle "look-for" tool are captured in Figure 1.2. Keep in mind that each element might not be evident during a short visit to a math class. Some things may or may not be present due to when the class is visited or how long the visit lasts. Elements could also be removed to improve the functionality of the tool. The tool is available to download. The reproducible also has space to record feedback that should be shared with the teacher.

Figure 1.2 • Productive Struggle Classroom Walkthrough Tool

- ❑ High-quality task is the centerpiece of the lesson.
- ❑ Lesson is focused on understanding the math rather than simply finding a solution.
- ❑ The classroom shows evidence of resources that support student struggle.

- ❑ Students have access to instructional resources and mathematics tools.
- ❑ Students are purposefully engaged in the task.
- ❑ Students attempt to overcome challenges.
- ❑ Students ask questions about the task or how they might proceed.
- ❑ Students support one another without taking over.

- ❑ Teacher circulates to monitor student thinking, progress, and struggle.
- ❑ Teacher pauses engagement to discuss strategies, representations, or questions.
- ❑ Student solutions are discussed and explored when the task is debriefed.
- ❑ Debriefing discussion highlights challenges and strategies used for overcoming them.
- ❑ Students reflect on struggle at the end of the lesson.

This tool is available to download from
resources.corwin.com/productivemathstruggle

The walkthrough tool has other useful applications. It could be used as a framework for supporting professional learning sessions in conjunction with the actions presented in this book. It could support the efforts of a professional learning community (PLC). It might also be used for teachers in peer-to-peer visits to guide what they might look for and support feedback specifics.

As with other look-for tools, productive struggle look-fors come with some considerations. First and foremost, look-fors or walkthroughs are generally intended as brief (10 to 15 minutes) visits to classrooms. It's possible that certain behaviors aren't evident due to the brevity of the visit or the segment of the lesson that was observed. Multiple measures are needed. Even so, a look-for could be modified to focus on something specific. It could simply look to see if students ask questions or work productively with classmates. Or, it could look for ingredients necessary for productive struggle including the use of a high-quality task or anchor charts that suggest to students what they might do when they struggle.

Knowing behaviors of productive struggle and actively looking for them during instruction declares that it is valued.

Whole School Agreement About Productive Struggle

Everyone in a school must be on the same page when it comes to struggle. Karp, Bush, and Dougherty (2016, p. 61) state that a whole school agreement for mathematics is crucial, as students need to hear common and consistent messages from those who support their learning. This idea easily applies to how productive struggle is supported and valued. Teachers, administrators, and support staff such as paraeducators, instructional coaches, or interventionists must believe that mathematics is more effectively taught when students engage in productive struggle.

This means it is important for the adults in a building to take time for conversation about struggle. Staff must talk about what struggle is, what it looks like when it is both unproductive and productive, what causes each, and how to respond to students who are struggling to help them struggle productively. Then, every adult needs to commit to these beliefs through their daily actions and conversations with students. If common values and beliefs regarding teaching and learning mathematics are not mutually upheld, the mixed messages students might receive as a result may have a large effect on how they view struggle.

Let's consider an example about when a whole school agreement has not been established. In this example, a teacher has worked hard to cultivate a classroom learning community that values the ideas of not giving up and working through a task even if it appears to be initially difficult. One day, the teacher provides a prompt to a particular student to get her thinking about what strategy she might utilize, and walks away so the student can engage with the task independently. Then, entering the classroom, the paraeducator sees the student struggling, approaches her to help, and ends up providing a step-by-step explanation of how to get to the answer. While the assistance

was well intended, the paraeducator's intervention undermines the teacher's intention to provoke productive struggle and inadvertently sends a negative message about the value of struggle. Yes, it may be a challenge to get every adult in a building on the same page, but doing so is necessary to support students as they learn to engage in and value productive struggle.

Struggle and a Growth Mindset

As we noted earlier in the chapter, there is a strong connection between productive struggle and a growth mindset. Indeed, a growth mindset is necessary for struggle to be productive. Students must believe wholeheartedly that they can learn mathematics through dedication and hard work (Dweck, 2006). To engage in struggle, students must value the tenets of a growth mindset. They must know that struggle is an important component of the learning process. But believing and doing are different. They must also know that they are capable of and are developing the skills to work through struggle when they believe in themselves and persevere.

What beliefs about struggle are prevalent in your classroom, school, or community?

As a teacher, it is essential that you too believe each and every student in your classroom is capable of learning mathematics and that they can do so through struggle and effort. Your comments and actions should foster an environment where growth mindsets are promoted and nurtured. We should value and select tasks that will challenge students to think and make sense of mathematics.

It is not enough to simply have or advocate for a growth mindset. It must come to life in math lessons and be evident in your work in the classroom. It must be put into action. Planning and teaching math is not a predetermined endeavor but one that requires growth through learning, experience, exploration, and reflection. Mistakes are opportunities to learn and grow. Student struggles are opportunities for teachers to learn and grow as well. Through this learning process, how you and your students define a good lesson might shift from the idea that everyone "gets it" easily the first time. This shift comes to life by actions described in this book.

Schools that value growth mindsets are positioned to promote productive struggle as a part of a healthy learning environment. They are a home to teachers and students who feel free to make mistakes in the name of learning. They communicate not only that it's okay to make a mistake, correct it, and move on but also that mistakes are to be fostered, explored, and used. Staff in these schools exchange ideas about how they consistently value, react to, and make use of student mistakes so that students don't get mixed messages from class to class or grade to grade.

Moving From Unproductive to Productive Beliefs About Struggle

Armed with knowledge of and value of productive struggle, teachers are better able to counter unproductive beliefs that can interfere with learning and productive struggle. They can help the school community make key shifts to more productive beliefs. Below are "popular" unproductive beliefs about struggle and the shifts in thinking that need to occur:

- Unproductive belief: *Students who are good at math don't struggle.*
- Productive belief: *All students should have opportunities to struggle in math.*

Figure 1.3 • Struggle Shifts

Unproductive Belief	Productive Belief
Students who are good at math don't struggle.	**All students should have opportunities to struggle in math.**
Everyone struggles for one reason or another at some time or another. It's true that some students don't experience struggle until middle school or even high school. Many of those students form deeply held beliefs about the "weakness" of struggle. When they encounter it for the first time—and they will—they begin to question perceptions of themselves. Struggle can be considerably unnerving if it hasn't been a regular part of learning math in the past. This means it is important for each and every student, regardless of what they know or what they can do mathematically, to have an opportunity to struggle.	
Students need to master basic skills before we ask them to struggle with challenging problems and applications.	**Basic skills can be taught and reinforced through application to new problems.**
This belief is similar to the assumption that students can't work with fractions until they know their basic facts. While knowing the basic facts frees up cognitive load, allowing students to focus solely on learning the new content, students without the basic facts can still learn new content. It *might* take longer as working memory has the extra burden of shifting focus from new content to fact recall and back again. Still, it isn't okay to deprive students of struggle because they haven't fully mastered a skill. It is analogous to not letting six-year-olds play T-ball because they don't know all of the rules of baseball or because they can't pitch, field, or throw with proficiency.	
Low-performing students aren't capable of dealing with struggle.	**All students are capable of struggle.**
Any student is capable of struggle if armed with strategies for working through it. In fact, "low" students are familiar with not knowing something immediately. Struggle can be much more frustrating, even defeating, for those who are used to having math come very easily to them. Most importantly, all students, whatever their current ability level, are not only capable of but entitled to high-quality instruction. We can't reserve this, including struggle, for higher-achieving students while having a goal of "basic skills" for others.	
Struggle undermines confidence.	**Struggle builds confidence.**
Struggle can undermine confidence if the wrong ideas about what it means to do and be successful with math are deeply held. In most, if not all, other aspects of life, struggle and overcoming struggle empowers individuals and strengthens confidence. Comparing math to such life situations is a useful strategy for helping students change their fixed ideas.	

Unproductive Belief	Productive Belief
Math is straightforward. You either get it or you don't. Struggle signals a problem.	**Learning math is complex. Answers to math problems might be right or wrong, but the solutions and strategies can vary.**
Perceptions of math as black and white or right and wrong are narrow and misguided. Learning and doing math can be complex. Struggles with learning mathematics do not necessarily signal a problem of any kind; in fact, they lead to deep understanding. There are many pathways to an answer.	
Using tools, such as calculators, is cheating.	**Tools support students as they navigate struggle.**
There are different types of tools. Some tools, including manipulatives, drawings, and graphs, help students see and make sense of math concepts. Other tools, including calculators, number charts, and fact charts, help students calculate accurately. These tools ensure accuracy. They alleviate distractions and frustrations that can cripple productive struggle. Students shouldn't be denied access to learning new content if they need these tools. We should also take action to help them calculate without the tools.	
Struggle takes too much time.	**Struggle results in learning that sticks.**
Struggle does take time. Meaningful learning takes time. More may be covered without spending time to struggle, discuss, and unpack student thinking. However, those topics may not be covered well and ultimately lead to frequent reteaching. The initial instruction, gobs of practice, and need for reteaching as well as review before tests may actually take more time than what would have been needed to foster productive struggle with the skill or concept. Learning with struggle helps learning "stick" and students retain it more.	
A class can't get through struggle without strong student role models in the classroom.	**Students benefit not from seeing others do it right but from observing and discussing strategies and reactions with other students.**
It is helpful for students to have classmates who exhibit productive struggle behaviors. This helps individuals develop their own strategies and reactions. However, this is not to say that a class has to have a group of students who are "good at math" so that others can learn math from them, or that a class has to have a group of "strong" students so that discussion can take place. While peer-to-peer learning is valuable, having students simply show other students how to do math is another take on simply giving students information instead of helping students develop their own understanding. These actions still lead to student reliance on someone else doing math for them instead of doing it themselves.	
Faster is better. Struggle is a signal that one isn't good at math.	**People who are good at math can struggle with different skills or concepts.**
Faster is not better in math (Boaler, 2015a, 2015b). It takes time for students to develop their own understanding. Engineers, electricians, and accountants, among people in countless other fields, struggle with problems and calculations for an extended amount of time. Professional mathematicians may spend months or even years working to find a solution to a single problem. Struggle is unpredictable. It may come when it is least expected. The amount of time needed changes from topic to topic and student to student.	
Some things in math, like word problems, are okay to struggle with but simple facts and procedures shouldn't be a struggle.	**All aspects of math are open to struggle and deserve to be learned and done well, no matter how long it takes.**
Students can struggle with all sorts of topics in math. Word problems and applications may seem more challenging because students must think and reason. But the same can be said for decomposing numbers, comparing fractions, and graphing lines. It is dependent upon the type and depth of understanding targeted for those concepts.	

KEY TAKEAWAYS ABOUT ACTION 1: VALUE PRODUCTIVE STRUGGLE

Teachers and students value what matters. Struggle matters. It cannot be said enough to students, family, and colleagues. Message abundance and action affirm value. The action of valuing productive struggle includes these big ideas:

- Productive struggle can be thought of as *purposefully reacting to an unclear challenge so that progress is made or learning advanced.*
- Struggle is not always productive. It is important to learn how to avoid engaging students in unproductive and even destructive struggle, while promoting struggle that is productive.
- There are student behaviors and teacher behaviors indicative of productive struggle in math.
- Evidence of productive struggle can be collected during observations, walkthroughs, and peer visits. Collecting evidence helps teachers and principals measure the frequency and quality of productive struggle in the classroom.
- Productive struggle is more likely to be realized in individual classrooms when the whole school and all stakeholders value it.
- Unproductive beliefs about teaching and learning math, including productive struggle, must be countered.

The bottom line about all of these beliefs is that values are based on who we are. Values are grown from experiences, interactions, perceptions, and feelings. Values are grounded in our identities and the communities in which we thrive. So, what is our math identity? What is our students' math identity? How do we build a community of identities that values and embraces productive struggle? In Action 2, you will learn how to explore these questions in order to establish a foundation for productive struggle in your classrooms.