# *The Common Core Companion* at a Glance

**Suggested Materials:** Provides teachers with a list of materials that will be helpful in introducing the concepts in this domain. "Reproducible" indicates that there is a Reproducible that you can use to make multiple copies in the Resources section in the back of the book.

**SUGGESTED MATERIALS FOR THIS DOMAIN**

| K | 1 | 2 | |
|---|---|---|---|
| ✓ | ✓ | ✓ | Objects for counting such as beans, linking cubes, two-color counter chips, coins |
| ✓ | | | Five frames (Reproducible 1) |
| ✓ | ✓ | ✓ | Ten frames (Reproducible 2) |
| | ✓ | ✓ | Double ten frames (Reproducible 3) |
| ✓ | ✓ | ✓ | Hundreds chart (Reproducible 4) |
| ✓ | ✓ | ✓ | Dot cards (Reproducible 5) |
| ✓ | ✓ | | Numeral cards (Reproducible 6) |
| | ✓ | ✓ | Number line to 20 (Reproducible 7) |
| | ✓ | ✓ | Open number line (Reproducible 8) |
| ✓ | ✓ | ✓ | Part-Part-Whole chart (Reproducible 9) |
| ✓ | ✓ | ✓ | Place value chart (Reproducible 10) |
| ✓ | ✓ | ✓ | Various Dice (1–6, 1–10) |
| ✓ | ✓ | ✓ | Various Spinners (1–4, 1–5, 1–6, 1–10) |

## Operations and Algebraic Thinking

### Domain Overview

**KINDERGARTEN**
Students build upon their understanding of counting to develop meaning for addition and subtraction through modeling and representing problem situations, using concrete objects and pictorial representations. This domain comprises the major work of kindergarten and will be developed across the entire school year. Table 1 in the Resource section provides a detailed chart of addition and subtraction situations.

**GRADE 1**
As first graders continue to develop fluency with addition and subtraction, problem solving provides an opportunity for them to make sense of these operations using various situations and contexts. First graders extend their work from kindergarten by representing additional situations for addition and subtraction (Table 1). They also develop more sophisticated strategies for addition by counting on rather than starting with 1, for subtraction by counting back from a total (sum), and by composing and decomposing addends.

Note that in the early grades the term *total* is used rather than *sum* when referring to the answer in addition or the starting number in subtraction. This is intentional in order to avoid any confusion between *sum* and *some*, words that sound the same but have very different meanings.

**GRADE 2**
As students demonstrate understanding, skill, and ability to apply addition and subtraction to all problem situations, the range of numbers with which they work increases to 100. Problem situations include simple two-step problems for students to model and explore. Students extend their expertise with mental mathematics strategies (Table 2) initially using concrete materials and later as they continue to practice and become fluent with addition and subtraction facts including all facts through sums of 20.

This domain is not taught in isolation from the Number and Base Ten domain. Students work across domains to develop a deep understanding of addition and subtraction focusing on the instructional shifts of developing conceptual understanding, building skill and fluency, and applying addition and subtraction in problem contexts.

**Domain Overview:** Gives a brief description of the big ideas, allowing you to see how the mathematical ideas develop across grade levels.

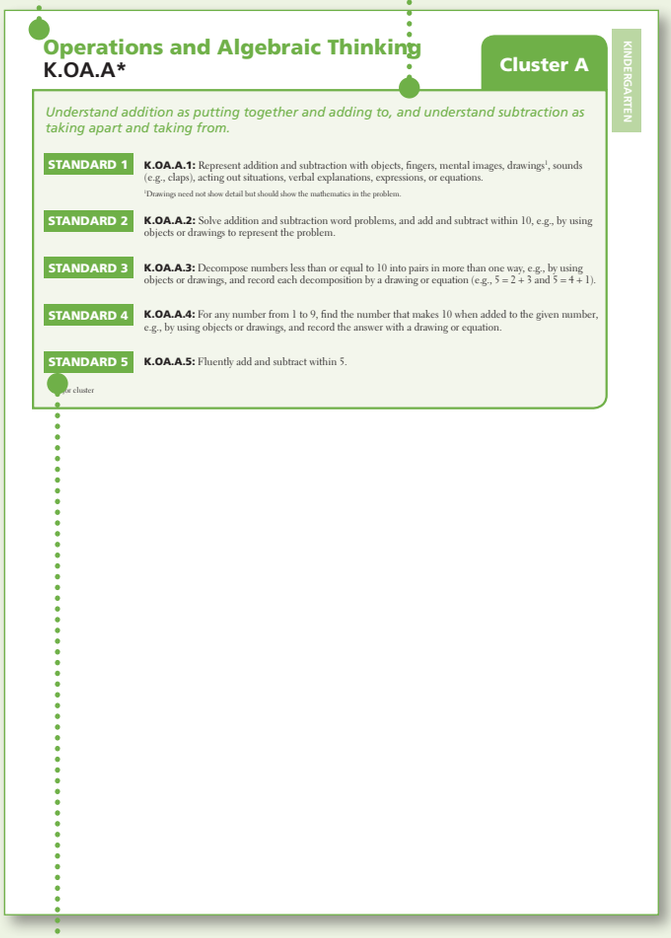
**Key Vocabulary:** Vocabulary included in the domain, noting the grade levels at which that term is used. This terminology can be used for building a word wall in the classroom. Students should be able to use these terms in talking about mathematics in discussions unless otherwise noted. Standard for Mathematical Practice 6 (Attend to Precision) calls for students to use mathematical terminology appropriately.

**Domain:** General mathematical topic for this group of standards.

**Cluster:** Statements that summarize related standards.

**KEY VOCABULARY**

| K | 1 | 2 | |
|---|---|---|---|
| ✓ | ✓ | ✓ | **add**—to combine or join together<br>*related words: add, and, plus, join, put together, (+)* |
| | ✓ | ✓ | *** associative property of addition**—an extension of the commutative property, to change the order and group 2 addends to find convenient sums (such as 10) in order to make the addition easier. Note that students do not use parenthesis at this level. The focus is on looking for sums of 10.<br>4 + 8 + 2 = 4 + 10 = 14 or 6 + 8 + 4 = 6 + 4 + 8 = 18 |
| | ✓ | ✓ | *** commutative property of addition**—reversing the order of the addends does not change the total (sum)<br>8 + 5 = 13 and 5 + 8 = 13; therefore, 8 + 5 = 5 + 8 |
| ✓ | ✓ | ✓ | **compare**—to look for similarities or differences among numbers |
| | ✓ | ✓ | **compose**—put a number together using other numbers<br>1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5, 1 + 2 + 3 + 4 are ways to compose 10 |
| | ✓ | ✓ | **decompose**—separate a number into parts using other numbers<br>8 can be decomposed into 4 + 4, 3 + 5, 2 + 2 + 2 + 2 |
| ✓ | ✓ | ✓ | **difference**—the amount by which one number is greater or less than another number. The difference can be found by subtracting, comparing, or finding a missing addend. |
| ✓ | ✓ | ✓ | **equal (=)**—same as in value or size |
| | ✓ | ✓ | **equation**—a mathematical sentence in which one part is the same as, or equal to, the other part |



**Operations and Algebraic Thinking**
**K.OA.A***

**Cluster A**

KINDERGARTEN

*Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.*

**STANDARD 1** **K.OA.A.1:** Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[1]Drawings need not show detail but should show the mathematics in the problem.

**STANDARD 2** **K.OA.A.2:** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

**STANDARD 3** **K.OA.A.3:** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

**STANDARD 4** **K.OA.A.4:** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

**STANDARD 5** **K.OA.A.5:** Fluently add and subtract within 5.

the cluster

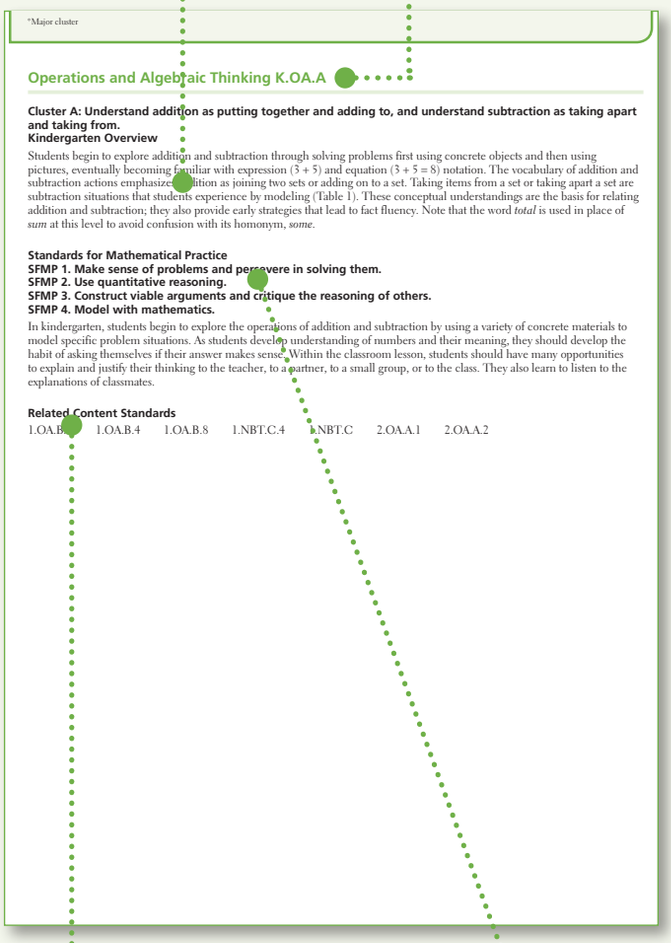
**Standards:** Mathematical statements that define what students should understand and be able to do.

K = Grade
OA = Domain
A = Cluster

Each cluster begins with a brief description of the mathematics in that cluster.

The standard as written in the Common Core is followed by an explanation of the meaning of the mathematics in that standard and what it looks like in the classroom.

An overview of actions the teacher might take in introducing and teaching the standard. This is not meant to be all inclusive, but rather to give you an idea of what classroom instruction might look like. We include illustrations of how to use materials to teach a concept where using models and representations is called for in the standard.



*Major cluster

**Operations and Algebraic Thinking K.OA.A**

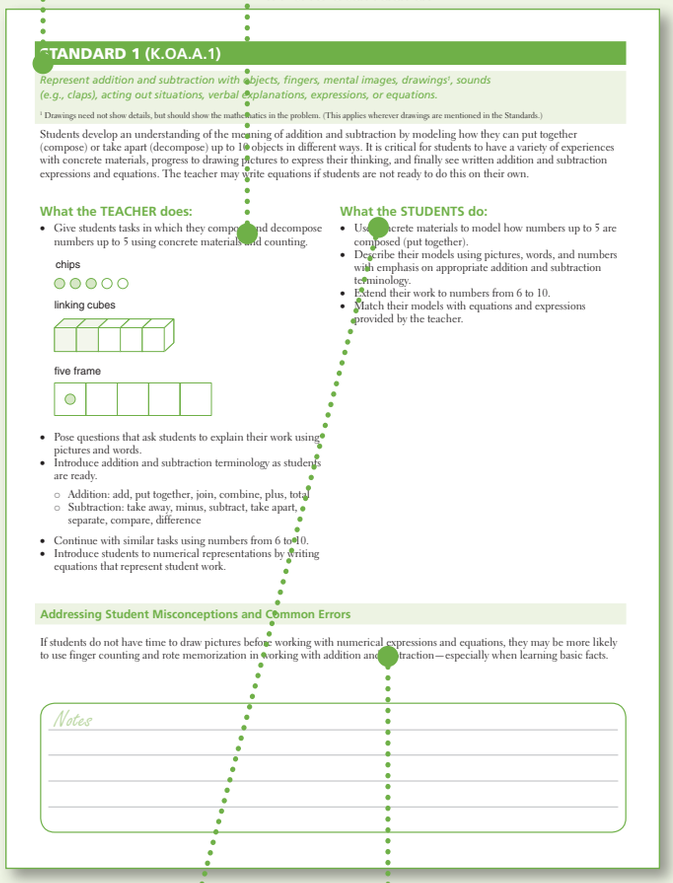
**Cluster A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**
**Kindergarten Overview**

Students begin to explore addition and subtraction through solving problems first using concrete objects and then using pictures, eventually becoming familiar with expression (3 + 5) and equation (3 + 5 = 8) notation. The vocabulary of addition and subtraction actions emphasizes addition as joining two sets or adding on to a set. Taking items from a set or taking apart a set are subtraction situations that students experience by modeling (Table 1). These conceptual understandings are the basis for relating addition and subtraction; they also provide early strategies that lead to fact fluency. Note that the word *total* is used in place of *sum* at this level to avoid confusion with its homonym, *some*.

**Standards for Mathematical Practice**
**SFMP 1. Make sense of problems and persevere in solving them.**
**SFMP 2. Use quantitative reasoning.**
**SFMP 3. Construct viable arguments and critique the reasoning of others.**
**SFMP 4. Model with mathematics.**

In kindergarten, students begin to explore the operations of addition and subtraction by using a variety of concrete materials to model specific problem situations. As students develop understanding of numbers and their meaning, they should develop the habit of asking themselves if their answer makes sense. Within the classroom lesson, students should have many opportunities to explain and justify their thinking to the teacher, to a partner, to a small group, or to the class. They also learn to listen to the explanations of classmates.

**Related Content Standards**
1.OA.B.3 1.OA.B.4 1.OA.B.8 1.NBT.C.4 1.NBT.C 2.OA.A.1 2.OA.A.2



**STANDARD 1 (K.OA.A.1)**

*Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.*

[1] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Students develop an understanding of the meaning of addition and subtraction by modeling how they can put together (compose) or take apart (decompose) up to 10 objects in different ways. It is critical for students to have a variety of experiences with concrete materials, progress to drawing pictures to express their thinking, and finally see written addition and subtraction expressions and equations. The teacher may write equations if students are not ready to do this on their own.

**What the TEACHER does:**

- Give students tasks in which they compose and decompose numbers up to 5 using concrete materials and counting.

chips

linking cubes

five frame

- Pose questions that ask students to explain their work using pictures and words.
- Introduce addition and subtraction terminology as students are ready.
  - Addition: add, put together, join, combine, plus, total
  - Subtraction: take away, minus, subtract, take apart, separate, compare, difference
- Continue with similar tasks using numbers from 6 to 10.
- Introduce students to numerical representations by writing equations that represent student work.

**What the STUDENTS do:**

- Use concrete materials to model how numbers up to 5 are composed (put together).
- Describe their models using pictures, words, and numbers with emphasis on appropriate addition and subtraction terminology.
- Extend their work to numbers from 6 to 10.
- Match their models with equations and expressions provided by the teacher.

**Addressing Student Misconceptions and Common Errors**

If students do not have time to draw pictures before working with numerical expressions and equations, they may be more likely to use finger counting and rote memorization in working with addition and subtraction—especially when learning basic facts.

*Notes*

**Related Content Standards:** Provides a list of standards connected to this topic in other grade levels, as well as standards in this grade level related to this topic. Consider the related standards as you plan your instruction for each cluster.

Although it is likely you will use a variety of standards of mathematical practice in teaching each cluster, this section gives examples of how you might incorporate some of the practices into your instruction on this topic.

Some examples of what students might be doing as they explore and begin to understand the standard. Again, this is not intended to be directive but rather to frame what student actions might look like.

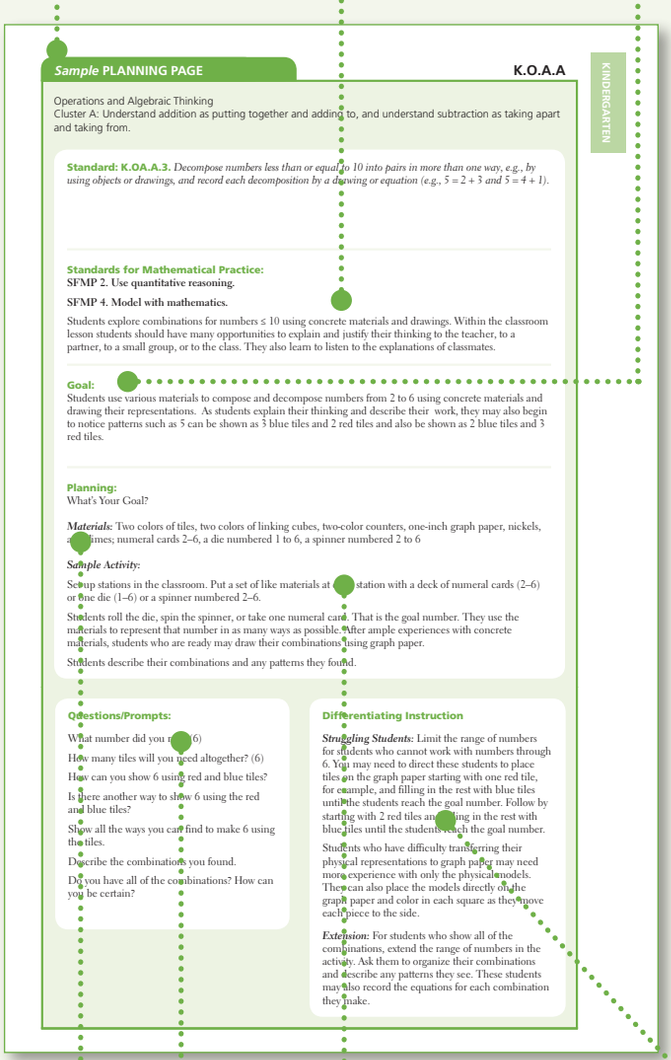
Each standard concludes with a description of student misconceptions and common errors and suggested actions to address those misconceptions.

We have provided a complete sample planning page for one individual standard at the end of each grade level. It is not meant to be a final lesson plan, but rather to identify the areas you should consider while planning your lessons for the standards.

Defines the purpose of the lesson and shows how it connects to previous (and future) ideas.

Identifies the mathematical practices that might be emphasized in this lesson.

A planning template is provided at the end of each cluster. This template is provided for your use as you consider instructional actions around a particular standard. You might want to make copies of this page and use them for each standard within the cluster. This is not intended to be an all-inclusive lesson plan. Rather, it gives you a place to record your thoughts about teaching a mathematical topic as you read the standard.



*Sample* PLANNING PAGE

K.O.A.A

KINDERGARTEN

Operations and Algebraic Thinking
Cluster A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

**Standard: K.OA.A.3.** *Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).*

**Standards for Mathematical Practice:**
**SFMP 2. Use quantitative reasoning.**
**SFMP 4. Model with mathematics.**

Students explore combinations for numbers ≤ 10 using concrete materials and drawings. Within the classroom lesson students should have many opportunities to explain and justify their thinking to the teacher, to a partner, to a small group, or to the class. They also learn to listen to the explanations of classmates.

**Goal:**
Students use various materials to compose and decompose numbers from 2 to 6 using concrete materials and drawing their representations. As students explain their thinking and describe their work, they may also begin to notice patterns such as 5 can be shown as 3 blue tiles and 2 red tiles and also be shown as 2 blue tiles and 3 red tiles.

**Planning:**
What's Your Goal?

***Materials:*** Two colors of tiles, two colors of linking cubes, two-color counters, one-inch graph paper, nickels, pennies; numeral cards 2–6, a die numbered 1 to 6, a spinner numbered 2 to 6

***Sample Activity:***

Set up stations in the classroom. Put a set of like materials at each station with a deck of numeral cards (2–6) or one die (1–6) or a spinner numbered 2–6.

Students roll the die, spin the spinner, or take one numeral card. That is the goal number. They use the materials to represent that number in as many ways as possible. After ample experiences with concrete materials, students who are ready may draw their combinations using graph paper.

Students describe their combinations and any patterns they found.

**Questions/Prompts:**

What number did you roll? (6)

How many tiles will you need altogether? (6)

How can you show 6 using red and blue tiles?

Is there another way to show 6 using the red and blue tiles?

Show all the ways you can find to make 6 using the tiles.

Describe the combinations you found.

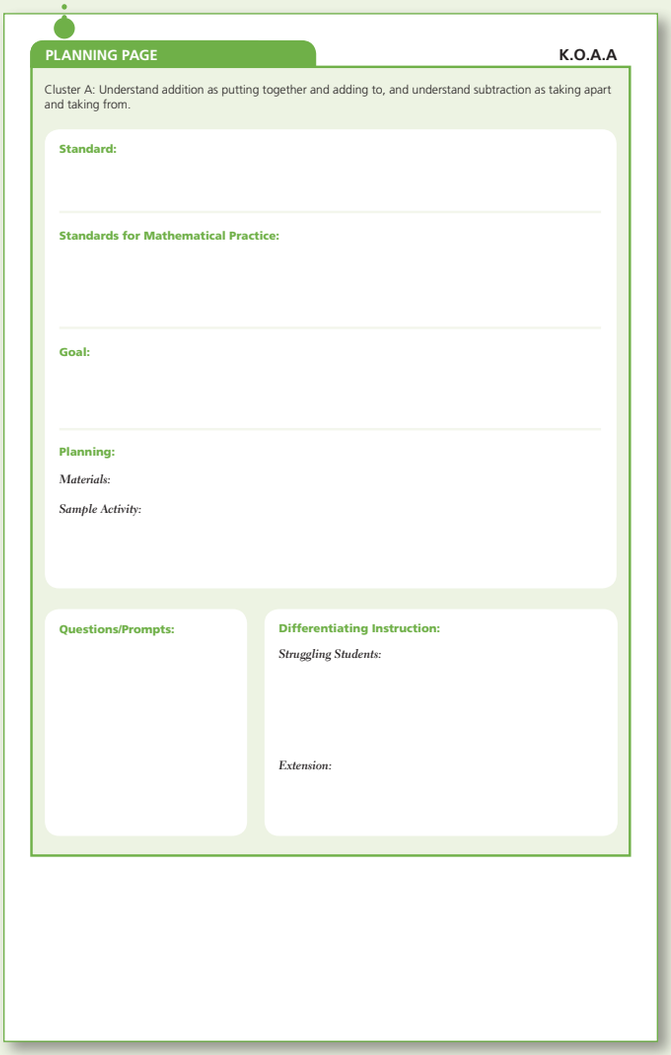
Do you have all of the combinations? How can you be certain?

**Differentiating Instruction**

***Struggling Students:*** Limit the range of numbers for students who cannot work with numbers through 6. You may need to direct these students to place tiles on the graph paper starting with one red tile, for example, and filling in the rest with blue tiles until the students reach the goal number. Follow by starting with 2 red tiles and filling in the rest with blue tiles until the students reach the goal number.

Students who have difficulty transferring their physical representations to graph paper may need more experience with only the physical models. They can also place the models directly on the graph paper and color in each square as they move each piece to the side.

***Extension:*** For students who show all of the combinations, extend the range of numbers in the activity. Ask them to organize their combinations and describe any patterns they see. These students may also record the equations for each combination they make.



PLANNING PAGE

K.O.A.A

Cluster A: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

**Standard:**

**Standards for Mathematical Practice:**

**Goal:**

**Planning:**

***Materials:***

***Sample Activity:***

**Questions/Prompts:**

**Differentiating Instruction:**

***Struggling Students:***

***Extension:***

Lists the materials that will be used to teach this standard.

Includes directions for the task students will complete.

It is important to anticipate student thinking throughout the lesson. Think about the questions or prompts you might give to help build student understanding and encourage student thinking.

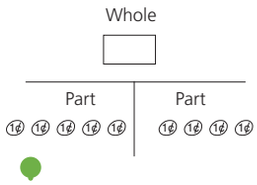
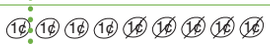
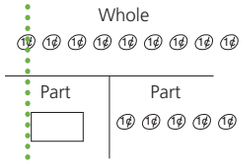
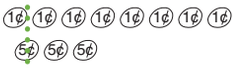
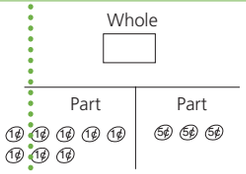
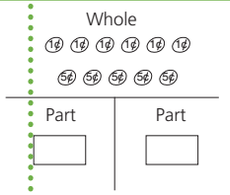
Provides an area where teachers can identify how they might adjust the lesson to (1) address the needs of students who are struggling and (2) extend the lesson for students who demonstrate understanding of the mathematics.

In the resources section you will find an overview of the standards for mathematical practice and what each standard means for students, the effective teaching practices from NCTM's Principles to Actions, and an overview of each practice for teachers to consider and implement; Table 1 which provides problem solving situations, Table 2 which provides strategic competencies for students, and Table 3 which scaffolds and includes modeling examples for the operations of addition and subtraction across grades K-2; and Reproducibles for some of the materials recommended for each grade level.

Various problem situations for addition and subtraction with suggested grade levels.

An example of a problem that exemplifies the situation.

Equation(s) that represent the situation.

| SITUATION | PROBLEM | PHYSICAL MODEL | PART PART WHOLE | EQUATION(S) |
|---|---|---|---|---|
| *Add to—result unknown*<br><br>Grades K, 1, 2 | Frank had 5 pennies. Mark gave him 4 more. How many pennies does Frank have? |  |  | **5** + **4** = ☐ |
| *Take from—result unknown*<br><br>Grades K, 1, 2 | Frank had 9 pennies. He spent 5 pennies on a jawbreaker. How many pennies does he have left? |  |  | **9** – **5** = ☐ |
| *Put together take apart—total unknown*<br><br>Grades K, 1, 2 | Anna has 8 pennies and 3 nickels. How many coins does she have? |  |  | **8** + **3** = ☐ |
| *Put together take apart—addends unknown*<br><br>Grades K, 1, 2 | Anna has 11 coins. Some are pennies and some are nickels. How many pennies and how many nickels could Anna have? | **Show table with different combinations of pennies and nickels that will total 11 (use illustrations of coins)**<br>**Pennies** / **Nickels**<br>1. 1¢ / 10. 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢<br>2. 1¢ 1¢ / 9. 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢<br>3. 1¢ 1¢ 1¢ / 8. 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢<br>4. 1¢ 1¢ 1¢ 1¢ / 7. 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢<br>5. 1¢ 1¢ 1¢ 1¢ 1¢ / 6. 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ |  | **1** + **10** = **11**<br>**2** + **9** = **11**<br>**3** + **8** = **11**<br>**4** + **7** = **11**<br>**5** + **6** = **11**<br>**6** + **5** = **11**<br>**7** + **4** = **11**<br>**8** + **3** = **11**<br>**9** + **2** = **11**<br>**10** + **1** = **11** |

A physical model that students might use to represent the situation.

An alternate model (part whole/bar model).

This table includes a scaffolded list of concepts and skills that students should develop in K–2.

| Grade Level | Description | Example | Model | Other strategies/models |
|---|---|---|---|---|
| K 1 2 | 1 digit + 1 digit (basic facts with sums to 20) | 9 + 7 | | Make a ten<br>Decompose 7 = 1 + 6<br>Add 9 + 1 + 6 = 16 |
| 1 | 2 digit + 1 digit (no regrouping) | 23 + 6 | | Benchmark numbers<br>Open number line<br>Basic facts |
| 1 | Add multiples of 10 | 30 + 50 | | Place value<br>Basic facts<br>Open number line |

An example of place value based models

Other strategies that support student understanding

A variety of reproducibles can also be downloaded from the companion website at **resources.corwin.com/mathematicscompanionK-2** for student use.

# Hundreds Chart

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# Quick Reference Guide

## KINDERGARTEN

### Counting and Cardinality K.CC

**A. Know number names and the count sequence.**

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence.
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.

**B. Count to tell the number of objects.**

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

**C. Compare numbers.**

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
7. Compare two numbers between 1 and 10 presented as written numerals.

### Operations and Algebraic Thinking K.OA

**A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

1. Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds, acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation.
4. For any number from 1 to 9, find the number that makes 10 when added to the given number.
5. Fluently add and subtract within 5.

### Number and Operations in Base Ten K.NBT

**A. Work with numbers 11-19 to gain foundations for place value.**

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

### Measurement and Data K.MD

**A. Describe and compare measurable attributes.**

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.

**B. Classify objects and count the number of objects in each category.**

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

### Geometry K.G

**A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional or three-dimensional.

**B. Analyze, compare, create, and compose shapes.**

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and other attributes.
5. Model shapes in the world by building shapes from components and drawing shapes.
6. Compose simple shapes to form larger shapes.

**Note**: More detail and examples from individual standards can be found in the complete standards document available at www.corestandards.org

## GRADE 1

### Operations and Algebraic Thinking 1.OA

**A. Represent and solve problems involving addition and subtraction.**

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[1]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

**B. Understand and apply properties of operations and the relationship between addition and subtraction.**

3. Apply properties of operations as strategies to add and subtract.
4. Understand subtraction as an unknown-addend problem.

**C. Add and subtract within 20.**

5. Relate counting to addition and subtraction.
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

**D. Work with addition and subtraction equations.**

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

### Number and Operations in Base Ten 1.NBT

**A. Extend the counting sequence.**

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

**B. Understand place value.**

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a "ten."
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

**C. Use place value understanding and properties of operations to add and subtract.**

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

### Measurement and Data 1.MD

**A. Measure lengths indirectly and by iterating length units.**

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

**B. Tell and write time.**

3. Tell and write time in hours and half-hours using analog and digital clocks.

**C. Represent and interpret data.**

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

### Geometry 1.G

**A. Reason with shapes and their attributes.**

1. Distinguish between defining attributes versus nondefining attributes; build and draw shapes to possess defining attributes.

2. Compose two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.[1]

3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

## GRADE 2

### Operations and Algebraic Thinking 2.OA — 2.OA

**A. Represent and solve problems involving addition and subtraction.**

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

**B. Add and subtract within 20.**

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

**C. Work with equal groups of objects to gain foundations for multiplication.**

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

### Number and Operations in Base Ten — 2.NBT

**A. Understand place value.**

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

   a. 100 can be thought of as a bundle of ten tens—called a "hundred."

   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2. Count within 1000; skip-count by 5s, 10s, and 100s.

3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

**B. Use place value understanding and properties of operations to add and subtract.**

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.

### Measurement and Data — 2.MD

**A. Measure and estimate lengths in standard units.**

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

3. Estimate lengths using units of inches, feet, centimeters, and meters.

4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

**B. Relate addition and subtraction to length.**

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, and equations with a symbol for the unknown number to represent the problem.

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram.

**C. Work with time and money.**

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

**D. Represent and interpret data.**

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.

## Geometry 2.G

**A. Reason with shapes and their attributes.**

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

### Standards for Mathematical Practice (K–2)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

For a downloadable version of this Quick Reference Guide, visit the companion website at **resources.corwin.com/mathematicscompanionK-2**

# The Common Core Mathematics Companion: The Standards Decoded, Grades K–2

# The Common Core Mathematics Companion: The Standards Decoded, Grades K–2

## What They Say, What They Mean, How to Teach Them

Linda M. Gojak

Ruth Harbin Miles

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# Contents

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—Linda M. Gojak

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—Ruth E. Harbin Miles

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# Letter to K–2 Teachers

Dear K–2 Teachers,

*The Common Core Mathematics Companion: The Standards Decoded, Grades K–2* is designed to support you as you help your students learn what they truly need to know and be able to do. This book includes critical ideas of K–2 mathematics and is intended to be your content guide. A brief overview for each standard along with effective teaching practices, mathematics vocabulary, suggested models, manipulatives, representations, and ideas for each standard are included. The book is intended to help you make sense of and decipher the content standards and Standards for Mathematical Practice that were developed to promote student achievement. These standards have the potential to change traditional classroom instruction across the United States. This is significant, as the content standards will help ensure students deeply understand the mathematics they are expected to learn. The content standards are the foundation for developing a rigorous, relevant, and coherent mathematics curriculum for every student and will help ensure all students are ready for their futures including college and the workforce.

The Common Core State Standards for Mathematics promote conceptual understanding and reasoning as well as skill proficiency. Included in the common core document are the standards, clusters, and domains. The domains for K–2 mathematics include Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry. Clusters are groups of related standards for each domain. The standards under each domain and cluster stress conceptual understanding of key ideas and procedural skills and applications and are inherently connected to other standards within and across grade levels.

Also included in the Common Core State Standards document are eight Standards for Mathematical Practice. These standards inform us of what our students should be able to do in mathematics in order to learn with understanding. The "student" standards are (1) Make sense of problems and persevere in solving them. (2) Reason abstractly and quantitatively. (3) Construct viable arguments and critique the reasoning of others. (4) Model with mathematics. (5) Use appropriate tools strategically. (6) Attend to precision. (7) Look for and make use of structure. (8) Look for and express regularity in repeated reasoning. When students are actively involved in using the standards, they are learning meaningful, high-quality mathematics.

We suggest that you work with your grade level colleagues and use this book when you are studying the standards to decide upon the sequencing and clustering of the standards as well as the selection of appropriate instructional resources. Be sure to study the content for the grade before and after the one you teach, so you will understand what students should have learned and what they will be learning the next year. Keep in mind that implementation of the standards and practices is a process and may take time to do well. Your devotion to teaching the standards will make a difference for students who will be learning to think, reason, and apply the mathematics you have taught them.

Sincerely,

Linda M. Gojak

Ruth Harbin Miles

# Letter to Elementary School Principals

Dear Elementary School Principals,

A leader must clearly explain content and help teachers understand that student success and achievement is the goal for implementing the Common Core Mathematics Content Standards and the Standards for Mathematical Practice. The role of the leader is to not only promote the standards but also to ensure the content standards are taught and the Standards for Mathematical Practice are achieved in every classroom. As an instructional leader, a principal must help teachers engage in professional learning to study both the content standards and the practice standards they will be teaching. Teachers will need guidance to understand the depth and the sequencing of each standard as well as the content before and after their grade levels. Leaders must help teachers understand that the Common Core Mathematics Standards have the prospect of ensuring equity and access to high-quality mathematics for every student.

The Common Core Mathematics Standards define what students should understand and be able to do in Grades K–2. Implemented properly, these standards will lay the foundation for concepts and skills students will be expected to know in the later Grades 3–12. Included in the common core mathematics document are the standards, clusters, and domains. The domains for K–2 mathematics include the content standards of Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry. The standards under each domain stress conceptual understanding of key ideas and continually spiral back to the organizing principles. Clusters are groups of related standards for each domain.

Also included in the Common Core State Standards document are eight Standards for Mathematical Practice. These standards inform us of what our students should be able to do in mathematics in order to learn with understanding. The student standards are (1) Make sense of problems and persevere in solving them. (2) Reason abstractly and quantitatively. (3) Construct viable arguments and critique the reasoning of others. (4) Model with mathematics. (5) Use appropriate tools strategically. (6) Attend to precision. (7) Look for and make use of structure. (8) Look for and express regularity in repeated reasoning. When students are actively involved in using the standards, they are learning meaningful, high-quality mathematics.

*The Common Core Mathematics Companion: The Standards Decoded, Grades K–2* is designed to support teachers in their learning and implementation of the Common Core Mathematics Standards. The book focuses on the critical ideas of K–2 mathematics including a meaningful explanation for each standard along with effective teaching practices and learning activities. Mathematics vocabulary and suggested teaching materials are highlighted for each standard. The book is not only a reference but also a guide to deciphering the standards.

Elementary schools, professional learning communities, and individual classroom and special education teachers will all have different knowledge, various skills, and distinct ideas about the Common Core Standards. A leader may wish to supply every teacher with a personal copy of the book for use as a school-wide initiative or book study. Such a study will help improve both content knowledge and understanding of the mathematics teachers are expected to teach. Providing the opportunity for teachers to engage and use the book in grade level planning with colleagues will allow teachers to dig deeply into the standards. Use of this resource will add cohesiveness and consistency ensuring all K–2 students will benefit from similar instruction. Be sure to invite teachers to bring this resource to all planning and professional development work. You may even want teachers to start or end a meeting with a lesson they've planned that is based upon the suggestions and strategies found in this reference guide. As a result of the book study, *The Common Core Mathematics Companion: The Standards Decoded, Grades K–2* will influence professional practice at both the classroom and school level and will help transform instruction.

Sincerely,

Linda M. Gojak

Ruth Harbin Miles

# Introduction

## A Brief History of the Common Core

Contrary to popular belief, academic standards are not new. In fact they have been around for more than twenty-five years. The first set of curriculum specific standards, *The Curriculum and Evaluation Standards for School Mathematics*, was released by the National Council of Teachers of Mathematics (NCTM) in 1989, followed by an updated set of standards, *Principles and Standards for School Mathematics*, in 2001. Both of these documents provided a vision for K–12 mathematics by grade level band. They also formed the foundation for most states' grade level standards.

In April 2009, the National Governors Association and the Council of Chief State School Officers met to discuss the creation of the Common Core State Standards Initiative. The purpose of such an initiative was to develop a set of common standards across states in order to balance the quality of mathematics instruction and learning. Following that meeting, the process of writing the common core standards began. The standards development team, led by William McCallum, Phil Daro, and Jason Zimba, included mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process included open invitation for feedback, not only from mathematics educators and associations including NCTM but also from the general public. This feedback was considered, and much of it was incorporated into the final document, which was released in June 2010. Following the release of the standards, individual states went through their own processes for reviewing, adopting, and, if necessary, ratifying the adoption of the Common Core State Standards.

## The Common Core State Standards for Mathematics

"The Common Core State Standards are a clear set of shared goals and expectations for the knowledge and skills students need in English language arts and mathematics at each grade level so they can be prepared to succeed in college, career, and life." (www.corestandards.org/about-the-standards/frequently-asked-questions/#faq-2303)

The Common Core State Standards for Mathematics (CCSSM) includes two critical components of learning mathematics. The content standards explicitly outline the mathematics we want students to know and be able to do at each grade level. The content standards of the common core are fewer in number than most previous state standards; at the same time, the expectation is that students will develop a deeper understanding of that content so less time is spent on reteaching from year to year. Additionally, the standards were carefully constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, you will find that the standards in Grade 1 develop from the mathematical work that students have completed in kindergarten. Similarly, the standards in Grade 2 develop from work completed in Grade 1. Thus, it is important for teachers to be knowledgeable of the standards not only at the level they are teaching but also at the preceding grade level and the next grade level.

The second group of standards, the Standards for Mathematical Practice, describes the habits of mind students should develop as they do mathematics. These eight standards are the same across all grade levels, K–12. As teachers plan mathematics lessons, they should consider how students will use the practices in learning and doing mathematics.

The Common Core Standards *are not* a curriculum. Decisions about mathematics programs, textbooks and materials, sequencing topics and units, and instructional frameworks are left for local or state school districts to make. They do not tell teachers how to teach. It is important to remember that they describe what students need to know and be able to do. Schools and teachers know best how to help students be successful with both the content and the practice standards.

The Common Core State Standards *do not* dictate specific assessments. Some states will be using assessments developed by *Partnership for Assessment of Readiness for College and Careers* (PARCC) or *Smarter Balanced Assessment Consortium* (SBAC, or Smarter Balanced). Others will develop and use their own assessments. Other facts and information can be found at http://www.corestandards.org.

## Instructional Shifts

While the standards do not call for a particular instructional model or philosophy, they are based on the best of existing standards. What is different is that they call for specific instructional shifts: **focus, coherence**, and **rigor**.

**Focus:** The content standards call for greater focus on fewer topics. An examination of the mathematics standards of high-performing countries indicates that fewer, more focused topics at a grade level allow students to deepen their understanding of the mathematics and gain a stronger foundation for an ongoing study of mathematics. Within the standards, the major mathematical work of each grade level has been identified (www.corestandards.org). That means that not all of the content within a grade is emphasized equally in the standards. The list of content standards for a grade is not linear, nor is it a checklist. Some clusters require greater emphasis than others. They take more time for students to master with depth of understanding. The major work of K–2 includes addition and subtraction—concepts, skills, and problem solving; and place value. This means the majority of instructional time in K–2 (65% to 85%) should be spent on these mathematical topics. This does not mean that other standards should be skipped. Rather, the supporting standards should be taught to connect mathematical ideas to the essential standards. The additional standards provide students with experiences that will be foundational to work in future grades. Neglecting material will leave gaps in student skill and understanding.

**Coherence:** Many of us learned mathematics as a set of disconnected topics, with much of our skill based on tricks ("ours is not to reason why, just invert and multiply") or mnemonic devices (Please Excuse My Dear Aunt Sally). In reality, mathematics is a coherent body of knowledge made up of topics that are connected and build on each other. The call for coherence in the content standards ensures that there are carefully constructed progressions from grade to grade so students build new understandings on the foundations laid in previous years. Each standard is not a new topic, but an extension of previous learning. In addition to the progressions across grade levels, the standards incorporate specific connections within a grade level. For example, as students learn the topics of addition and subtraction in the primary grades, the meaning of these operations is consistently connected to applying these understandings and skills to solving problems.

**Rigor:** The final instructional shift, rigor, refers to how to support students in developing deep understanding of each standard. Understanding does not develop by assigning more worksheets or more difficult examples and problems. Rather, it calls for instructional practice that balances conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts.

The following descriptions of each component of rigor come from the standards document, which can be found at www.corestandards.org.

*Conceptual understanding:* The standards call for conceptual understanding of key concepts, such as place value. Students must be able to access concepts from a number of perspectives in order to see mathematics as more than a set of rules or procedures.

*Procedural skills and fluency:* The standards call for speed and accuracy in calculation. Students must practice core skills, such as basic facts and addition/subtraction computation, in order to have access to more complex concepts and procedures. Fluency is built upon conceptual understanding and, with young children, through the development of ideas through representations using concrete materials, pictures, numbers, and words.

*Application:* The standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

## Major Work of K–2

To help drive the development of the content standards, between 65% and 85% of instructional time should focus on the major work for each grade level. (For K–2, the amount of time should be closer to 85%.) Areas of major work include:

**Kindergarten**—number ideas and counting, understanding addition as putting together and subtraction as taking from or taking apart, number facts with sums to 5, and work with numbers 11–19 to form the foundation of place value.

**Grade 1**—represent and solve problems with addition and subtraction, understand properties of addition and subtraction, fluently add and subtract within 10, work with sums to 20, extend the counting sequence to 120, understand foundations of place value using those ideas to add and subtract and measure lengths indirectly and by repeating length units.

**Grade 2**—represent and solve problems with addition and subtraction, fluently add and subtract within 20, understand place value, use place value and properties to add and subtract, measure and estimate lengths in standard units, and relate addition and subtraction to measuring.

Additional information on the focus for each grade level can be found in the resource section at the end of this book.

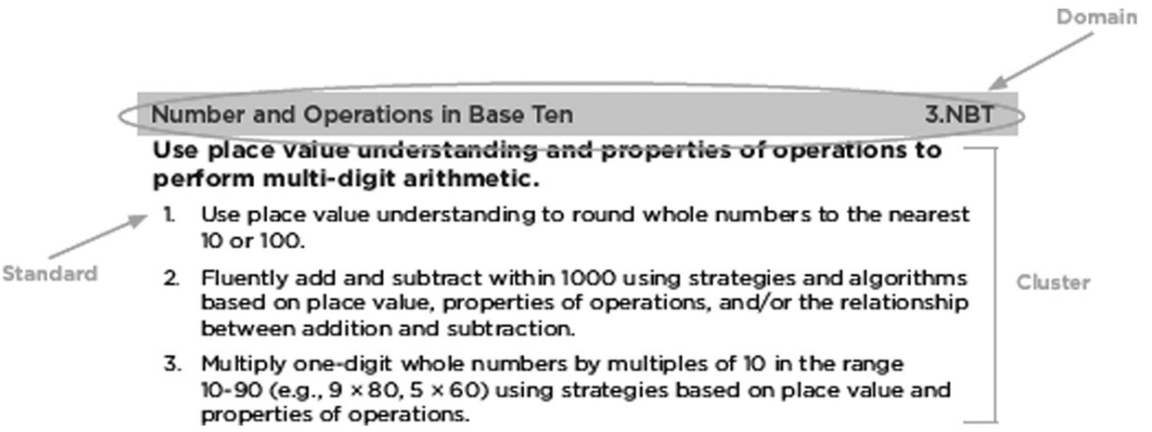
## Common Core Word Wall

The language of the common core differs from traditional standards. Familiarity with section names and their functions will help you to make the best use of the Common Core State Standards.

**Standards** define what students should understand and be able to do.

**Clusters** summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.




*Source*: Common Core State Standards for Mathematics (www.corestandards.org)

As districts develop units of study for a grade level, careful consideration should be given to the order of, and the connections among, topics and standards. For example, as kindergarten students are learning to rote count within a range of numbers (20 to 40), K.CC.A.1, they will also be counting objects to develop an understanding of number within a different range (1 to 10), K.CC.B.4a, at the same time.

The Common Core State Standards for Mathematical Practice describe eight habits of mind teachers must incorporate into classroom instruction to develop depth of understanding of critical mathematical concepts in their students. The mathematical practices are not intended to be taught in isolation but should be integrated into daily lessons. Some lessons may focus on developing one or two of these standards, while others may incorporate seven or all eight standards. Note that you do not "teach" these standards. Rather, they are the type of mathematical thinking and doing that we want students to practice as they are developing mathematical understanding.

Throughout each domain, we have included examples of mathematical practice that can be used in each cluster. These are not meant to limit lessons to using only those practices. They are examples of key practices that should be included in lessons around that particular cluster. It is likely that you will use all of the practices throughout the cluster and domain.

These eight practices, briefly explained here, are essential for student success. If students are actively engaged in using the eight practices, they are learning rigorous, meaningful mathematics.

## SFMP 1. Make sense of problems and persevere in solving them.

Students work to understand the information given in a problem and the question that is asked. They plan a solution path by choosing a strategy they can use to find a solution and check to make sure their answer makes sense. At the K–2 level, students will likely use manipulatives and pictures to conceptualize and solve a problem. Students should be encouraged to find multiple approaches to solve problems.

## SFMP 2. Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They develop operational sense by associating contexts to numbers (thinking about 4 + 7 in a context of putting together 4 pencils and 7 pencils) and associating mathematical meaning to given contexts (eating 4 of 11 strawberries can be expressed as 11-4). Students at the primary level often need to manipulate concrete objects to make meaning of numbers and find solutions.

## SFMP 3. Construct viable arguments and critique the reasoning of others.

K–2 students should have opportunities to explain their thinking, justify, and communicate their conclusions both orally and in writing. Listening to others and finding how their strategies are similar may take prompting questions from the teacher such as "Why do you think that works?" or "How is your method the same as . . .?" Mathematical discussions should be a common expectation in mathematics lessons. It will take time for students to become comfortable explaining their thinking, and this will develop over time. Explaining one's thinking helps to develop deeper conceptual understanding.

## SFMP 4. Model with mathematics.

Students use various representations, models, and symbols to connect conceptual understanding to skills and applications. Students in the early grades represent mathematics situations using objects, pictures, numbers, and words. Problem-solving strategies, such as draw a picture, make a list, or write an equation, have explicit connections to representations and models and can be developed at the same time.

## SFMP 5. Use appropriate tools strategically.

Students consider the available tools when solving a mathematics problem. In Grades K–2, students will begin with manipulatives before they draw pictures and move to pencil and paper. Students in K–2 also begin to think about appropriate measurement units, moving from informal to standard units to measure various items.

## SFMP 6. Attend to precision.

Students communicate precisely with others. K–2 students explain their thinking using the appropriate mathematical vocabulary. At this level, students are beginning to use mathematical symbols that should explicitly connect to vocabulary development.

## SFMP 7. Look for and make use of structure.

Students look closely to find patterns and structure in their mathematics work. For example, primary students begin to develop methods to compose and decompose numbers as well as recognize properties of operations. Place value and the commutative property are especially important concepts to develop at this level, as they are foundational to developing operation sense and skill.

## SFMP 8. Look for and express regularity in repeated reasoning.

Students notice if calculations are repeated and begin to make generalizations. By recognizing what happens when adding and subtracting tens, students develop a deeper understanding of place value in the early grades and later extend this understanding to what happens when adding and subtracting hundreds. Although this standard mentions shortcuts, it should be noted that shortcuts are only appropriate when students discover them through making generalizations and understand why they work.

## Effective Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outlines eight effective teaching practices every teacher should incorporate into their instruction to guarantee student achievement. These eight research-informed practices, briefly explained here, provide a foundation for effective common core mathematics teaching and student learning.

1. **Establish mathematics goals to focus learning.**

Establishing learning goals sets the stage and helps to guide instructional decisions. Teachers must keep in mind what is to be learned, why the goal is important, where students need to go (the trajectory), as well as how learning can be extended. Students must clearly understand the purpose of each lesson beyond simply repeating the standard.

2. **Implement tasks that promote reasoning and problem solving.**

Implementing tasks that promote reasoning and problem solving provides opportunities for students to engage in exploration and encourages students to use procedures in ways that are connected to conceptual understanding. The tasks teachers choose should be built on current student understandings and have various entry points with multiple ways for the problems to be solved.

3. **Use and connect mathematical representations.**

Using and connecting representations leads students to deeper understanding. Different representations, including concrete models, pictures, words, and numbers, should be introduced, discussed, and connected to support students in explaining their thinking and reasoning.

4. **Facilitate meaningful mathematical discourse.**

Facilitating meaningful student mathematical conversations provides students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. Talking and sharing aloud can advance the mathematical thinking of the whole class.

5. **Pose purposeful questions.**

Posing purposeful questions reveals students' current understanding of a concept and encourages students to explain, elaborate, and clarify thinking. Asking good questions makes the learning of mathematics more visible and accessible for student examination.

6. **Build procedural fluency from conceptual understanding.**

Building procedural fluency from conceptual understanding based on experiences with concrete representations allows students to flexibly choose from a variety of methods to solve problems.

7. **Support productive struggle in learning mathematics.**

Supporting productive struggle in learning mathematics is significant and essential to learning mathematics with understanding. Productive struggle allows students to grapple with ideas and relationships. Giving young students ample time to work with and make sense out of new ideas is critical to their learning with understanding.

8. **Elicit and use evidence of student thinking.**

Eliciting and using evidence of student thinking helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. Formative assessment through student written and oral ideas are excellent artifacts to assess student thinking and understanding.

More information on these practices can be found in Table 5 in the Resources section.

## How to Use This Book

The purpose of this book is to help teachers more deeply understand the mathematical meaning of each cluster and standard within the five domains of Grades K–2. We want this book to be your toolkit for teaching the mathematics standards, and we have left ample space for you to take notes, add ideas, and note other resources you have found to be helpful.

You will find each part is made up of one domain and begins with an overview of how the domain progresses across kindergarten, first grade, and second grade. A list of helpful materials, reproducibles, and key vocabulary from the domain is included in the overview as well.

We track each domain across kindergarten, first grade, and second grade, introducing each cluster and the standards within that cluster. A description of the cluster and how the standards for mathematical practice can be incorporated into your teaching of the cluster concepts follows. Since the standards are intentionally designed to connect within and across domains and grade levels, a list of related standards is included in the cluster overview. We suggest that as you prepare work on a cluster, you look at these standards to have a better idea of the mathematics students learned in previous grades and where they are going in future grades. A list of all of the standards is found inside the front and back covers.

Each standard within a cluster is explained with an example of what the teacher does to work with that standard in the classroom followed by what the students do. It is important to note that many standards will take several days, and it is likely that you will be connecting across standards and domains as you teach for understanding.

Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each standard.

A sample planning page for each grade-level domain provides an example for planning instruction for one standard. Additionally, a template for planning instruction is included at the conclusion of each grade-level domain.

In the resources section, you will find Tables 1–5, which are fundamental to the Operations and Algebraic Thinking and Number and Operations in Base Ten domains. You will also find reproducibles for key materials. These are designed to be samples, and we encourage you to use them or redesign them to best meet the needs of your students. A list of our favorite resource books and high-quality online resources that are particularly useful to developing mathematical ideas in K–2 are also included in the resources section.

We believe that this can become your common core bible! Read it and mark it with questions, comments, and ideas. We hope that it will help you to use these standards and good teaching practice to lay the essential foundation that will ensure your students success not only in your grade but also in all of their future study of mathematics.

## Reflection Questions

1. How are the three instructional shifts called for by the common core similar to your current instructional practice? What is conceptual understanding? How is it different from procedural skills? What do you need to consider to teach for conceptual understanding? How can you connect conceptual understanding to help students develop procedural skills? How does the information in Table 1 on problem situations support the development of conceptual understanding?

2. The Standards for Mathematical Practice describe the habits of mind that students need for thinking about and doing mathematics. While not every standard will be in every lesson, select one standard at your grade level and consider some ways you can incorporate these practices in a lesson for that standard. How will these practices provide you with information about student understanding? How will this help you to better assess students? How will this information help you in planning lessons?

3. The Effective Teaching Practices from *Principles to Actions* (2014; NCTM) describe specific actions that teachers must consider in planning and implementing lessons and assessing student performance. How are these practices connected? Work with colleagues to plan a lesson that employs all of these practices. What needs to be considered as you consider goals for the lesson? How can you modify a traditional task so that it promotes reasoning and problem solving? What representations will help students more deeply understand the concept? What questions will you ask students? How will you connect the conceptual understanding to build procedural fluency? What questions will support students who are working to make sense of a new idea? What kind of information will you look for to help inform your instruction? (For more information on the Effective Teaching Practices, go to www.nctm.org.)

Part 1

# *Counting and Cardinality*

# Counting and Cardinality

## Domain Overview

**KINDERGARTEN**

Students enter kindergarten with a broad range of experiences with numbers. Some will be able to count by rote from 1 to 100 (or a subset of that range). Others may have limited experience with counting to 10. Keep in mind that the content standards identify what students should know and be able to do by the end of kindergarten. Therefore, you will need to scaffold individual standards to meet the needs of students. For example, it is likely that you will begin the school year focusing on rote counting (sequencing number names) to 20 and at the same time work only on counting physical objects to 5. By the end of the year, students should be able to successfully complete all of these standards.

## SUGGESTED MATERIALS FOR THIS DOMAIN

### K

| | |
|---|---|
| ✓ | Objects for counting such as beans, linking cubes, counter chips, coins |
| ✓ | Five frames (Reproducible 1) |
| ✓ | Ten frames (Reproducible 2) |
| ✓ | Double ten frames (Reproducible 3) |
| ✓ | Hundreds chart (Reproducible 4) |
| ✓ | Dot cards (Reproducible 5) |
| ✓ | Numeral cards (Reproducible 6) |

## KEY VOCABULARY Vocabulary should be explored with kindergarten students using pictures and visual representations.

### K

| | |
|---|---|
| ✓ | **add**—to combine or join together<br>*related words: add, and, plus, join, put together, (+)* |
| ✓ | **compare**—to look for similarities or differences among numbers or their size |
| ✓ | **count**—to say numbers in order; to assign a value to a group of items based on one-to-one correspondence |
| ✓ | **difference**—the amount by which one number is greater or less than another number. The difference can be found by subtracting, comparing, or finding a missing addend. |
| ✓ | **equal (=)**—same as in value or size |
| ✓ | **fewer than**—less than |
| ✓ | **five frame**—a graphic representation that is useful to help students to count, see number relationships, and learn basic facts (Reproducible 1) |
| ✓ | **greater than**—more than |
| ✓ | **hundreds chart**—a 10-by-10 grid with the counting numbers from 1 to 100 listed; used to develop and demonstrate patterns and strategies for counting, addition, subtraction, and place value |
| ✓ | ***** **numeral**—a symbol that represents a number;<br>3 is the numeral that represents a count of 3 objects |
| ✓ | **number**—a count or measurement |
| ✓ | **subtract**—to take one number away from another; to find the difference between two numbers<br>*related words: subtract, minus, take from, take apart (–)* |
| ✓ | **ten frame**—a graphic representation that is useful to help students to count, see number relationships, and learn basic facts (Reproducible 2) |
| ✓ | **total (sum)**—the result when two or more numbers are added together |

*Students are not responsible for these vocabulary words; however, they should understand the mathematical concept.

# Counting and Cardinality
# K.CC.A*

**Cluster A**

*Know number names and the count sequence.*

**STANDARD 1** **K.CC.A.1:** Count to 100 by ones and by tens.

**STANDARD 2** **K.CC.A.2:** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

**STANDARD 3** **K.CC.A.3:** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

*Major cluster

## Counting and Cardinality K.CC.A

**Cluster A: Know number names and the count sequence.**
**Kindergarten Overview**

This cluster is about rote counting. Students do not need to have an understanding of what the numbers mean or of place value within this cluster. They will work with those concepts in the Order and Algebraic Thinking (OA) and Number and Operations in Base Ten (NBT) domains. Once students can count, they begin to connect number words with counting quantities. This should be accomplished in small increments.

**Standards for Mathematical Practice**
**SFMP 6. Attend to precision**

Students are learning numbers by rote counting. Vocabulary development, especially for students who have not had previous experience, includes counting as well as learning number names as they count by ones and by tens.

**Related Content Standards**

1.NBT.A.1 2.NBT.A.2

*Notes*

## STANDARD 1 (K.CC.A.1)

*Count to 100 by ones and by tens.*

Students work over the course of the year to count from 1 to 100. They begin counting by ones, and as the range of numbers grows, they also count by tens. Although this standard includes recognizing numerals, it does not include reading or writing numerals.

### What the TEACHER does:

- Provide many opportunities for students to count. Begin with a small range of numbers, such as 1 to 10, and increase the range depending on student needs. As students begin to rote count fluently, introduce the numeral representations for each number name. Give students a variety of opportunities to match the numeral with the number name.
- Use a variety of nursery rhymes, children's books, and songs to help students associate number sequence with situations that are already familiar to them.
- Once students can count to a given number, use a section of the hundreds chart to help them recognize the numerals that represent these numbers. Use matching games and activities to help students connect the number name with the numeral. The goal for this standard is for kindergarten students to count to 100 by the end of the year. This should develop over time depending on the readiness of students.
- As students become fluent in rote counting by ones, introduce counting by tens (10, 20, 30). Using a hundreds chart, ask them to identify any visual patterns they see in the numerals, such as they all end in 0 or the first digit goes in order.

### What the STUDENTS do:

- Students begin by sequentially counting by ones. They start with a limited range of numbers and increase the range until they can count to 100.
- They begin to match the number name with the numeral. They play games and complete activities using numeral cards and portions of the hundreds chart to connect numerals with number names.
- Students count to 100 by tens. As they learn the numerals for the digits 0 to 9, they begin to recognize patterns in the written numerals.
- Motion with fingers on both hands to count by tens.
- Use dimes to count by tens; this is a good way to introduce coins to students when they are ready to understand that one dime can represent ten cents.

### Addressing Student Misconceptions and Common Errors

Students who confuse the sequence of numbers (ex. 1, 4, 7, 3, 9, 2), skip numbers (ex. 1, 2, 3, 5, 6, 7, 9 . . .), or repeat numbers (1, 2, 3, 4, 2, 3, 4) need more experience counting within a smaller range of numbers. Students should be fluent within a range before increasing the range.

Words for the teen numbers may be confusing since they do not follow the pattern of other decade numbers (ex. fourteen vs. twenty-four). Provide more practice with reciting teen numbers and connecting the number name with the written numeral.

Focus on oral patterns such as the sequence of the ones place digits in the twenties is the same as the sequence of the ones place digits in the thirties.

20, 21, 22, 23, 24, 25, 26, 27, 28, 29

30, 31, 32, 33, 34, 35, 36, 37, 38, 39

Notes

## STANDARD 2 (K.CC.A.2)

*Count forward beginning from a given number within the known sequence (instead of having to begin at 1).*

Once students are fluent at counting beginning with 1, they begin to work on counting forward from a number other than 1 within a given range. This is a prerequisite skill for counting on as students begin to work with addition. This standard does not include writing numerals, which will follow at a later time.

### What the TEACHER does:

- Give the students a start number greater than 1 and ask the students to count on to that number within a range of numbers. For example, give a starting number of 5 and ask the students to count to 10. As in the previous standard, this is rote counting using number names. Understanding *number* as a count of objects is not necessary within this standard.
- While students are increasing the range of numbers to which they are counting, they are beginning to work on the standards within K.OA and K.NBT domains using lesser numbers.

### What the STUDENTS do:

- Students begin to rote count from a number other than 1 (counting on) without having to go back and start at 1.
- Given the number 3, the student should be able to continue the count (4, 5, 6) without starting from 1. Complete similar examples within a given range of numbers. Although recognition of written numerals may help students to count on, it is not essential. The expectation for this standard is rote counting.

### Addressing Student Misconceptions and Common Errors

Students who struggle with developing this standard, particularly with numbers greater than 10, should master counting within a sequence before counting forward from a number in that sequence. For example, students should be able to rote count to 20 before they are expected to count on from 8. Begin with smaller numbers and progress to greater numbers. Limit how far you want students to count and then increase the range.

Notes

## STANDARD 3 (K.CC.A.3)

*Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).*

Students recognize and write the numerals 0 to 20. Begin with the single-digit numerals and represent the number of items in a set with the appropriate numeral. Additionally, given a numeral, students can represent that number of items in a set.

### What the TEACHER does:

- Provide students with a variety of opportunities to recognize written numerals from 0 to 9. This can be accomplished with numeral cards, a deck of cards with both the numeral and a picture of the count of that many objects (Reproducible 6).
- Begin with a small range of numbers (0, 1, 2, 3). As students recognize those numerals with ease, add more cards to the range.


- Show students a collection of items and have them match the appropriate numeral card with the set.
- As students begin to recognize numerals, provide practice writing the numerals, using various modes including writing the numerals in the air as you model, writing numerals on large chart paper with a paint brush, writing in sand or shaving cream, or tracing the numerals on paper. Student readiness will vary with the development of eye-hand coordination and small motor skills.
- Teach this standard together with K.CC.B.2 so that as students count objects, they match the number of objects in a set with the numeral. Focus on activities that connect the concept of a specific quantity of objects and how they are represented by a number. The numeral is the written representation for that number.
- Start with a small range of numbers and continue with counting to 20, using a variety of objects to count including five frames, ten frames, and double ten frames. Include opportunities to count throughout the day such as counting steps, counting the number of students buying lunch, or counting the number of students who walk to school. Ask students to recognize and write numerals for the numbers they use throughout the day.

### What the STUDENTS do:

- Recognize numerals from 1 to 20.
- Match a collection of items with the appropriate numeral.
- Over time, write numerals from 1 to 20. Readiness to write the numerals will vary with the development of eye-hand coordination and small motor skills.
- Represent sets of objects with numerals after they have had experience with CC. Cluster B: Count to tell the number of objects.
- Use a variety of modalities to trace numerals in the air, in sand, on paper.

### Addressing Student Misconceptions and Common Errors

It is common for kindergarten students to invert or reverse numerals. With additional experience, most children will self-correct. Give children opportunities to have a variety of kinesthetic experiences to form numerals (write numerals in sand, rice, etc.) before they use paper and pencil.

# Counting and Cardinality
# K.CC.B*

**Cluster B**

*Count to tell the number of objects.*

**STANDARD 4** **K.CC.B.4:** Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

c. Understand that each successive number name refers to a quantity that is one larger.

**STANDARD 5** **K.CC.B.5:** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.

*Major cluster

## Counting and Cardinality K.CC.B

**Cluster B: Count to tell the number of objects.**
**Kindergarten Overview**

Students move from rote counting to finding the number of objects in a set. Cardinality refers to the actual count or number of items in a set. This cluster connects to the previous cluster. As students show proficiency rote counting within a range of numbers, for example, 1 to 10, they can begin to find the number of objects in a set within that range. While working within clusters A and B, it is important for students to connect the physical objects (3 counters) with the oral number word (*three*) and the numeral (3). Students should begin with counting physical objects, progress to pictures, and then connect the physical representations to the numeral.

**Standards for Mathematical Practice**
**SFMP 4. Model with mathematics.**
**SFMP 6. Attend to precision.**

Students continue to develop counting skills extending rote counting to actually counting concrete objects. They begin to develop the idea of one-to-one correspondence as they realize one number name goes with each item. Counting sequentially, starting with 1 and later counting by adding one to the number of items in a collection, helps students to know number names and the correct order of numbers as they match each count with one item.

**Related Content Standards**

K.CC.C.6 K.CC.C.7 1.NBT.A.1 2.NBT.A.3

*Notes*

## STANDARD 4 (K.CC.B.4)

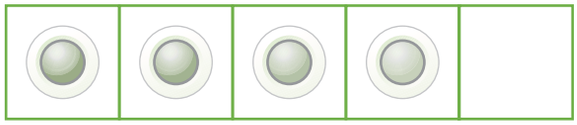
*Understand the relationship between numbers and quantities; connect counting to cardinality.*

a. *When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.*
b. *Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.*
c. *Understand that each successive number name refers to a quantity that is one larger.*

### What the TEACHER does:

- Provide opportunities for students to count using a variety of objects such as buttons, counters, shells, coins, and dot cards.
- Ask student to count objects, beginning with a smaller range of items and increasing as students count accurately.
- After students have counted items placed in organized arrangements (straight line, circle), arrange objects randomly.
- Use five frames to model linear representations of object and to help students begin to see patterns that make 5 (Reproducible 5).

**Show representations of 4 items**



- Ask, "How many are there?" to reinforce that the last number name tells the count of items rather than the counting process itself.
- Use formative assessment protocols to be sure that students understand the last number said tells the number of items in a collection.
- Have students count 5 cubes. Add one more cube to the set and ask how many now? (6). Progress to a similar setup but do not add the cube . . . ask "How many will there be if I add one more cube?" This helps students to visualize the process.
- After many experiences, ask, "If you had 5 cubes and added one more, how many would there be?" without using materials. Begin with numbers 1–5 and then increase the range of numbers to 10. It will take time for students to develop this conceptual understanding, so this standard should be developed over several months.
- As students are ready, extend this work to 10 using ten frames (Reproducible 2).

### What the STUDENTS do:

- Say the number name in consecutive order as they point to each object. Some students may find it helpful to move the objects as they count.
- Start by counting objects that are in a straight line and then move to organized representations (ex. arrays, circles) and finally randomly arranged objects.
- Indicate by counting that the last number said tells the number of items.
- Count on to the original number of items in a set, first by adding one item and later by mentally counting up one.

### Addressing Student Misconceptions and Common Errors

Watch for students who find it confusing to say one number name with one object as they count (one-to-one correspondence). Begin with a smaller number of objects and model saying the number name as you physically move the object. Have students do the same.

Watch for students who double count an object. Physically moving the object and saying one number name for each object will help to reinforce one-to-one correspondence; that is, one object goes with one number name. Students may see 5 items spread out as different from 5 items close together. Students should physically move the objects matching one item from one set with one item from the other set to understand that the count of 5 remains the same no matter how the objects are organized.

## STANDARD 5 (K.CC.B.5)

*Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.*

This standard builds on the previous standards in this cluster. Students continue to count items in a set, using physical and pictorial representations. In addition, given a number, students count out that quantity of items. Numeral recognition is developed throughout this cluster, so students should also recognize a written numeral and count a number of counters given the number orally or given the written numeral. Provide a variety of concrete experiences before students draw pictures.

### What the TEACHER does:

- Provide students with a bag, box, or bucket of objects and ask them to count out a certain number of objects. For example, say, "Show me 5 buttons." Begin with numbers to 5 and extend the range to 10, 15, and 20 as students show skill counting out objects.
- Ask students to match numerals with the number of items in the set they have counted.
- Give students a numeral card and ask them to read the number. Students then count out that many items to represent the number.
- Give a drawing of countable items, such as flowers, teddy bears, or cars, then ask students to circle a number of items and write the numeral.

### What the STUDENTS do:

- Count out a number of items using a variety of concrete objects.
- Match a numeral card with the number of items in a set.
- Given a written numeral, count that number of items from a collection of items.
- Given a drawing of items such as flowers, teddy bears, or cars, circle a number of items and write the numeral.
- Draw a given number of items.

### Addressing Student Misconceptions and Common Errors

Some students may be able to match a quantity with a number (or numeral) but cannot produce that number of objects when given materials or asked to draw a picture. Looking for a specific quantity when given a choice of collections has a lower level cognitive demand (is easier) than having to produce a set of objects given a number. This standard will take time to develop.

How many peanuts? Versus Show me 5 peanuts

Notes

# Counting and Cardinality
# K.CC.C*

## Cluster C

*Compare numbers.*

**STANDARD 6** **K.CC.C.6:** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g, by using matching and counting strategies.[1]

[1]Include groups with up to 10 objects.

**STANDARD 7** **K.CC.C.7:** Compare two numbers between 1 and 10 presented as written numerals.

*Major cluster

## Counting and Cardinality K.CC.C

**Cluster C: Compare numbers.**
**Kindergarten Overview**

Students build on the work of the previous clusters to develop strategies to compare two concrete quantities and later connect that idea to comparing two number words and two numerals. The language of more than (greater than) and less than (fewer than) can extend to "how many more?". . . and "how many less?," which begins the concept of additive thinking (one more than, two more than, one less than, etc.). Developing this language and giving students a variety of experiences will lay a solid foundation for future work with addition and subtraction.

**Standards for Mathematical Practice**
**SFMP 4. Model with mathematics.**
**SFMP 6. Attend to precision.**

This will be students' first experience with comparing quantities. Precision with language is critical in this cluster. Scaffolding experiences that start by using concrete materials with obvious comparisons and honing in on quantities that get closer in size will provide students with the time needed to understand the concepts.

**Related Content Standards**

K.MD.A.2 1.NBT.B.3 2.NBT.A.4

*Notes*

## STANDARD 6 (K.CC.C.6)

*Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.*[1]

[1]Include groups with up to 10 objects.

Students begin to work with two sets of objects to compare the number in each set. Note the scaffolding of tasks in the descriptions that follow.

### What the TEACHER does:

- Provide activities for students to compare quantities to determine which has more and which has less. Make the two groups very obvious so counting isn't needed to determine which has more. The focus is on developing appropriate vocabulary including greater than and less than. For example, show students a large bag of popcorn and a small bag of popcorn and develop the concept and language of more and less. "The big bag has *more than* the little bag." "The little bag has *less than* the big bag."
- Show two bags that are the same size and begin to work with the idea of equal using the language of *same as*.
- Once comparison vocabulary has been developed, students compare the number of items in two sets of objects and determine which has more and which has less. It is helpful to compare two different types of items (blue chips and red chips, circles and triangles), so there is no confusion when students begin to compare. There is a hierarchy of strategies involved when comparing. Students should use the strategy that makes the most sense to them.

  - *Matching*: Line up the items in each set using one-to-one correspondence. Which set has more? (triangles). Which set has less? (circles). Asking questions like "how do you know?" starts to develop reasoning and mathematical arguments called for in the Standards for Mathematical Practice.

  - *Observation*: I see there are more triangles than circles. When students use this strategy, it is important for them to explain how they "see" more triangles than circles. While an acceptable strategy, it is often difficult for them to explain how they know.
  - *Take away or fair share*: Each time I take a circle, you take a triangle. When all of the circles are gone, there will still be some triangles. Follow up with questions such as "Are there more triangles or circles? How do you know? Are there fewer triangles or circles? How do you know? Which shape has more? Which shape has fewer?"
  - *Compare counts*: Students count the number in each group and compare the counts. For example, there are 3 circles and 5 triangles, so there are fewer circles than triangles because 3 is less than 5.
- Transition to situations in which students are comparing the number of like items. For example, compare 5 peanuts with 7 peanuts.
- Once students recognize sets that are greater than or less than, give them situations in which they identify how much more than or how much less than one set is compared to another set (with differences of 1, 2, and 3).
- Provide situations where students identify equivalent sets. This may be their first experience with equality. Using the term *same as* may be more meaningful than *equal to* in describing equivalent sets.

### What the STUDENTS do:

- Develop vocabulary of greater than (more than) and less than (fewer than) to compare the number of items in two sets.
- Use various strategies that make sense to compare items in two sets, including matching, observation, take away/fair share, counting.
- Explain their reasoning to show that one set has more or less than another.
- Identify how many more or how many fewer items one set has than another.
- Recognize two sets that have the same number of items using the description *same as*.

## Addressing Student Misconceptions and Common Errors

Students who have trouble with the vocabulary of comparison need more opportunities to compare obvious amounts and practice the different ways to describe the comparison. For example, there are more teddy bear counters than chips. There are fewer chips than teddy bear counters.

The strategies above are listed in order of development. Students who are struggling to compare the size of two sets of items should line them up with one-to-one correspondence. If they are unable to keep the objects lined up, provide a sheet of one-inch graph paper and keep the items small enough so one item can fit in each square. Ask questions such as, Which row has more? How do you know? Which row has fewer (less)? How do you know?

Keep the number of objects in each set within the range of student success and then build to using greater numbers of items. Continue giving students opportunities to describe their thinking and to use comparison vocabulary.

Notes

## STANDARD 7 (K.CC.C.7)

*Compare two numbers between 1 and 10 presented as written numerals.*

This is the culminating standard of the Counting and Cardinality domain because it requires students to synthesize all of the previous standards. Students must be able to count items in a group, recognize number words and numeral representations, compare two groups of objects to identify which is greater or less, and associate numbers with each set to begin understanding the abstract nature of comparing numbers given only the numerals.

### What the TEACHER does:

- Once students show proficiency with comparing sets of objects (up to 10), repeat the same activities with different materials or pictorial representations. Have students place the numeral card for each set next to the items.
- Ask students to describe the comparison. There are more triangles than circles, so 5 is more than 3. As with the previous standards in this domain, students should have a variety of experiences with concrete and pictorial representations and then make explicit connections to the number names and numerals. Include situations so all of the comparison vocabulary is developed.


### What the STUDENTS do:

- Given two sets of concrete materials, students label each set with the appropriate numeral.
- Students compare the number of items in each set using comparison vocabulary and then connect the comparison of physical objects to the number names in describing the comparison.

### Addressing Student Misconceptions and Common Errors

Since this standard requires facility with all of the previous standards in this cluster, students who cannot accurately compare the number of physical objects are likely to struggle with comparing the numbers written as numerals. These students need additional practice comparing sets of objects and describing their reasoning before working with the numerals. Modeling the transition between the vocabulary of comparing the count of physical objects and using the same vocabulary with the number of items will help students to practice the vocabulary of greater than (more than), less than (fewer than), and same as.

*Notes*

## *Sample* PLANNING PAGE

**K.CC.B**

Counting and Cardinality
Cluster B: Count to tell the number of objects.

**Standard: K.CC.B.5.** *Count to answer "how many" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.*

### Standards for Mathematical Practice:

**SFMP 4. Model with mathematics.** Student uses a variety of concrete objects to show a given count.

**SFMP 6. Attend to precision.** Students will use correct number names and written numerals to accurately sequence numbers as they count out the number of items in the set.

### Goal:

As students show proficiency with rote counting, for example, from 1 to 10, they can begin to find the number of objects in a set within that range (cardinality). It is important for students to connect the physical objects, with the number word and the numeral. Students should begin with counting physical objects, progress to pictures, and then connect the numeral to the physical representations.

### Planning:

***Materials:*** Counters including chips, buttons, shells, and the like; five and ten frames; numeral cards.

***Sample Activity:***

- Model counting the number of objects in a set of 3 chips. Count orally as you move the items.
- Provide each table with a collection of items to count (less than 6 to start). Let students take turns counting items.
- As students show accuracy with counting, increase the number of objects.
- Have students match the numeral card with the correct count.
- Students who can write numerals can label each collection by writing the numeral.

### Questions/Prompts:

Show me 10 counters.

How can you prove there are 10?

Match the numeral for 10 with the items you counted.

*If a student double counts an item, prompt him to say one number with each item he moves.*

Write the number to show how many you have counted.

### Differentiating Instruction:

***Struggling Students:*** Begin with numbers less than 6. Have students move objects as they count. Be sure they are moving one item with each number. Next, have students point to each item as they count. Do not have students write numerals until they can count accurately.

***Extension:*** Give the students the number and ask them to show that many items and, later, draw that many items. Let students model numbers with ten frames and double ten frames explaining different strategies to know the number. For example, I know there are 15 because 5 spots are empty.

## PLANNING PAGE

**K.CC.A**

Cluster A: Know number names and the count sequence.

**Standard:**

**Standards for Mathematical Practice:**

**Goal:**

**Planning:**

*Materials:*

*Sample Activity:*

**Questions/Prompts:**

**Differentiating Instruction:**

*Struggling Students:*

*Extension:*

## PLANNING PAGE

**K.CC.B**

Cluster B: Count to tell the number of objects.

**Standard:**

**Standards for Mathematical Practice:**

**Goal:**

**Planning:**

*Materials:*

*Sample Activity:*

**Questions/Prompts:**

**Differentiating Instruction:**

*Struggling Students:*

*Extension:*

## PLANNING PAGE

**K.CC.C**

Cluster C: Compare numbers.

**Standard:**

**Standards for Mathematical Practice:**

**Goal:**

**Planning:**

*Materials:*

*Sample Activity:*

**Questions/Prompts:**

**Differentiating Instruction:**

*Struggling Students:*

*Extension:*

## Reflection Questions: Counting and Cardinality

The order of these standards is not meant to be linear—in other words, you do not teach one standard and then move to the next. Rather, they connect within a cluster, across clusters, and across domains.

1. Look at the standards in Cluster A: Know number names and the count sequence. How are these standards related? Discuss how to organize these standards to help meet the individual needs of your students.

2. Cluster B: Counting to tell the number of objects addresses students' ability to count objects. How does this cluster build from Cluster A? How can you assess student needs to determine the range of numbers to work with, beginning with the standards in Cluster A and then moving to Cluster B? For example, some students may need to work on counting to 5; others may be ready to count to 10 or even 20.

3. The standards in Cluster C talk about early comparison of two quantities. What are some grade-level-appropriate activities that kindergarten students can do to begin to understand the meaning of greater than (more than), less than (fewer than), and same as (equal)?