# Preface

## WHAT THIS BOOK IS ABOUT

Since the publication of the first edition of this text, accountability for the achievement of high standards by educational systems was federally legislated in the form of required state-produced high-stakes assessments and improvement mandates by the No Child Left Behind (NCLB) legislation (U.S. Department of Education, 2001; Linn, Baker, & Betebenner, 2002). The legislation was initially responsive to heightened public attention and criticism of the problems of our educational systems. Not the least of that criticism was directed at the performance of our students in mathematics, especially when it was compared to that of students from other countries on international tests such as the Trends in International Mathematics and Science Study (TIMSS; National Center for Education Statistics, 2005). The NCLB legislation called for mandated assessments in literacy and mathematics from Grades 3–8 and identified specific sanctions for schools failing to show improvement in terms of adequate yearly progress. Although the tests are developed on a state level, the sanctions are powered by the withholding of federal funds.

A mixture of criticism and support continues to confront the assessments and their connected mandates. Most of the negative response is related to the sanctions imposed on poorly performing schools and the resulting stress on teachers and students. A recent report by the National Conference of State Legislatures (Dillon, 2005) says the law sets unrealistic expectations and defies commonsense notions of how to rate schools. State lawmakers also cite the conflict of NCLB with other legislation that protects the disabled. There has actually been some backtracking on previously instituted state-generated actions and a push for more state control over NCLB criteria and sanctions. Unfortunately, despite evidence that the assessments have begun to improve student performance in some places, unresolved deficiencies or gaps in the overall achievement of specifically identified groups of students continue the calls for improvement (University of the State of New York, 2005).

Despite the varying opinions on the value of high-stakes assessments in the context of the stress and limitations they place on teachers and students and their possible misuse, it is my firm belief that assessment has a vital role in the educative process. It is most productive, however, when used as a tool through which the teacher manages instruction. When intensive professional development has accompanied careful curriculum construction and attention is paid to teacher ownership, assessments matched to that curriculum have contributed to improved student performance. I will address this issue further in Chapter 1 (for in-depth analyses, see Solomon, 1995, 2002, 2003).

Although there are still a number of possible, unproven reasons for the less-than-desired overall performance of U.S. students and the persistent gaps for certain groups of students, the first possibility that our educational community responded to was that our curriculum may have been an affecting factor. Comparisons of math curriculum in our country to that in countries more successful on the international assessments revealed that ours covered too many topics repetitively and lacked intensity and focus. The initial response to this possible reason for failure, therefore, was to develop standards that outlined the necessary curriculum. Standards, initially published by the National Council of Teachers of Mathematics (NCTM, 1989, 2000), were then individually adapted by states to create state standards documents.

The state standards documents have helped to remedy some of the deficiencies in the curriculum, but some early versions of the state standards have been recently criticized for lack of clarity and specificity (Klein et al., 2005). This book may provide some of what is missing. The first edition of *The Math We Need to Know and Do* was actually cited by the New York State Education Department as a key resource for its newly issued and much more explicit standards document.

My own observations of teachers in various states and socially different schools reveal that despite published curriculum documents, most of what is taught in schools today is still governed by published texts and workbooks. There is much that is worthwhile in today's textbooks for young learners; they provide drill and practice for operations and some good and varied application activities. New-generation technology like graphing calculators and computer software that allows for exploration and spatial problem solving, as well as access to data sets on the Internet, is even more hopeful if used properly. The deficiency in the technology and some texts lies in the fact that they do not clearly delineate for the teachers or the students what exactly one needs to know to be an effective quantitative problem solver. Nor do they help teachers understand and build upon what research has taught us about how learning happens. Frequently, our students learn how to do the procedures in the books without constructing mathematical concepts that may be generalized to novel or real-life problems. Curriculum documents published by State Education Departments try to define the skills and expectations but usually neglect to clarify the underlying or embedded mathematical concepts—and frequently are too general to override the day-to-day teacher-friendly comforts of text programs.

Pedagogical texts for teachers do this, but often they, too, neglect the concepts. Strangely, very old mathematics textbooks stated very clearly and simply what the necessary concepts were. *New Practical Arithmetic,* which was written by Benjamin Greenleaf (1872), brings the learner from the very elementary notations of single digit numbers all the way to cube roots and the applications of stocks, bonds, taxes, principal, and interest within 322 small (4" by 6") pages.

On these pocketbook-sized pages are 465 paragraph sections that include precise definitions and succinct statements of the concepts as well as limited exercises. What Greenleaf did establish, for his time, was a clear mathematical knowledge base.

Although the world has gained much new knowledge since 1872, and topics like probability, statistics, and mathematical modeling have to be added to help prepare our students for the technology-based modern world, the basic known content or knowledge base of elementary mathematics is not that different. The modern world, however, may not offer children the same kind of learning experiences. Hands-on real-life experiences are replaced by computer games, and automatized facts, drill, and practice by calculator computations. Nevertheless, our pedagogical knowledge base, especially our knowledge about how children learn, is much expanded. We use many new and better approaches to learning and teaching. Preoccupation with this new pedagogical knowledge has perhaps distracted us from the mathematics knowledge itself. For example, knowing that estimation skills and the ability to factor algebraic expressions with understanding and facility depend upon quick mental retrieval of multiplication facts can lead us to value the automatization of facts. Knowing that automatization may be easier at earlier developmental stages might encourage us to allocate that expectation to an early grade.

This book accepts the current climate of accountability by assessment and focuses on both the identification of the specific embedded concepts—*what we need to know*—and the matching skill expectations—*what we need to be able to do*—in order to apply and demonstrate what we know. Using current national and state standards as a guide, it covers these elements of mathematics content for Grades K–5, relates it to the current expanded pedagogical knowledge, and offers suggestions for instructional approaches and sequencing.

It is designed as a resource for teachers to use as they

- Plan curriculum for a school, particular grade level, or specific lesson
- Assess their students' knowledge, both formally and informally
- Respond to individual conceptual or procedural problems among their students
- Review their own mathematical concepts

Like Greenleaf, I have tried to be parsimonious with words. For in-depth discussions of the background research, readers can refer to the literature cited in the References. This is also not a mathematics textbook, although it will provide some illustrative activities for students. Instead, it will compensate for the missing components of recent texts and curriculum guides—statements of the very specific concepts and procedures embedded in mathematical knowledge. These are phrased succinctly and precisely in Chapter 2 as the *embedded concepts* (*What students need to know*) and articulated *skill or performance indicators (What students need to be able to do).* These tell us more precisely exactly what students need in order to solve the problems in their texts, the real world, and the state assessments. They tell us some of what students need to know in order to have life-long comfort and ease with new mathematical problems and to compete with others in a technological future. They will help teachers analyze whether their students have achieved the specifics of that knowledge and guide them in the correction of unsound or incomplete constructions of knowledge.

This book is a resource meant to be used by teachers in conjunction with other materials: texts, workbooks, manipulatives, and technology. It can also serve as an adjunct textbook for teachers-in-training—one that focuses more intensely on the content as it applies rather than generalizes the pedagogy. In order to accomplish our purpose of clarity in the presentation of the concepts, the embedded concepts and their matching skills and performance indicators are presented in numbered-table form in Chapter 2. Chapter 3 then provides correspondingly numbered suggestions for the matching instructional dialogue, manipulatives, and sample problems that can be used by the teacher to help develop the concept or skill. The problems are designed to develop the embedded concepts, but are also forms of "*proximal*" assessment. Proximal assessment is the informal form of assessment that teachers, who are close to those they assess, need to do in the classroom as they teach. With minor additions, however, the problems can be adapted for more formal forms of assessment.

There are clear purposes for this separation. Chapter 2 can be used to plan school curriculum from a multi-grade or single-grade perspective. Teachers can use it as a daily assessment check and lesson planning guide, and as an easy reference for a look back at the grounding concepts from previous grades. If further clarification is needed, or the teacher needs suggestions for how to scaffold the concept with dialogue and problems to solve, there is a simple cross-check to the more comprehensive Chapter 3. However, no single activity or set of activities is guaranteed to assure the new knowledge for all students. Just doing a prescribed activity is not enough: The embedded concept has to be constructed by the student and assessed by the teacher.

Although there are suggestions offered for the vocabulary and substance of the teacher-directed dialogues and peer interactive discourses that can help students construct new knowledge, this is far from a script. It is different from many curriculum guides produced by teachers in that it shows the sequential and specific development of concepts over the grades, rather than at a specific grade. The reason for this is so that teachers may check for prior knowledge and know where a particular concept can lead. It is hoped that this will make their curriculum more responsive to the individual differences among their students.

The concepts or content standards included are a composite from many sources. Many were identified over time by careful personal and shared collegial observation of students' thinking: from pre-kindergarten through graduate classes in math teaching methods. They reflect mathematics educators' most current research on how children learn mathematics as reported in the literature, but also pull from resources as disparate and remote as Piaget, Greenleaf, and a comprehensive curriculum guide published by the Baltimore schools in 1952 (Baltimore Public Schools, 1952). They are functionally based on the ideas and organization of the year 2000 versions of the standards for mathematics developed by the National Council of Teachers of Mathematics (2000) and other state and local agencies. These may be more comprehensive in terms of the pedagogy rationale and should be consulted in tandem with this book. Most of these documents are, however, less specific and organized about the content knowledge, particularly the embedded concepts and definitions. This leaves much for the teacher to provide. For example, the standards statements are often framed in terms of understanding, such as: "Students will understand the relationship between multiplication and division." Chapter 2 and certainly Chapter 3 are more explicit about what it is that the students need to understand. Concept statements and scaffolds for building understanding would include ideas such as,

> In multiplication we know the size of each group and the number of the groups we add repeatedly, but not the size of the whole. We multiply to find the whole. In division we know the whole and either the number of groups or the size of the group, but not both. We divide to find the size of each group or the number of groups.

This book will fill some of the gaps, but certainly not every possible construction of knowledge. Others may be identified or newly constructed by teachers as they begin to teach in a different way—with a clearly identified concept or construct in mind. If they provide opportunities for their students to reason and solve problems creatively, new concepts for both teacher and student may be intersubjectively (Lerman, 1996) constructed.

The book is presented in three chapters, which should be considered in sequence. Chapter 1 provides a rationale for the suggested learning approach and explains the organization and sequence of the following chapters. Chapter 2 provides the actual content standards in numbered table form, showing median grade-level expectations for concepts and skills or performance indicators and suggestions for mathematics language vocabulary and usage. The content standards are organized to agree with the organization of the NCTM standards with some exceptions that are explained in Chapter 1. Chapter 3 provides articulated illustrative activities and problems that can be used with students, either for concept development or assessment purposes. It also contains suggestions for using tools other than text materials: manipulatives, calculators, educational software and graphics programs (commercial and shareware), and Web sites.